

Assignment #5

Transition Systems and Linear Temporal Logic

Due Date: 12-02-2020 by 11:59pm

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The problems have been adapted from the book - Principles of model checking by Christel Baier and Joost-Pieter Katoen.

Remember the following notation:

\diamond	<i>eventually</i>
\square	<i>always</i>
\bigcirc	<i>next</i>
\cup	<i>until</i>
\neg	<i>negation</i>
\vee	<i>or</i>
\wedge	<i>and</i>

1 PROBLEM 1

(15 points - 2+2+2+3+3+3)

Consider the following transition system over the set of atomic propositions $\{a, b\}$:

Indicate for each of the following LTL formulae the set of states for which these formulae are fulfilled:

- (a) $\bigcirc a$
- (b) $\bigcirc\bigcirc\bigcirc a$

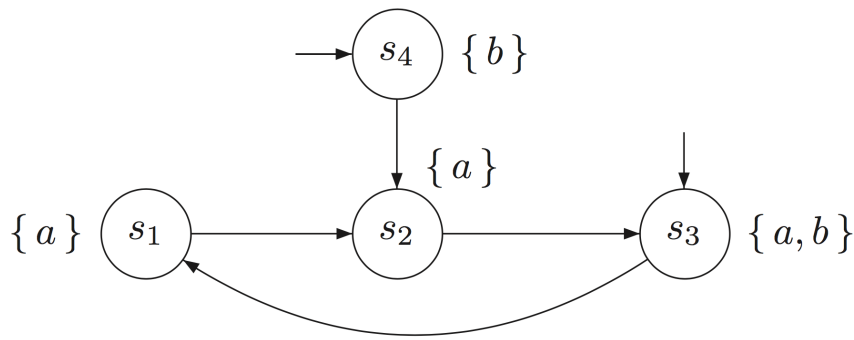


Figure 1.1:

- (c) $\Box b$
- (d) $\Box \Diamond a$
- (e) $\Box (b \cup a)$
- (f) $\Diamond (a \cup b)$

2 PROBLEM 2

(15 points - 2+2+3+2+3+3)

Consider the transition system TS over the set of atomic propositions $AP = \{a, b, c\}$:

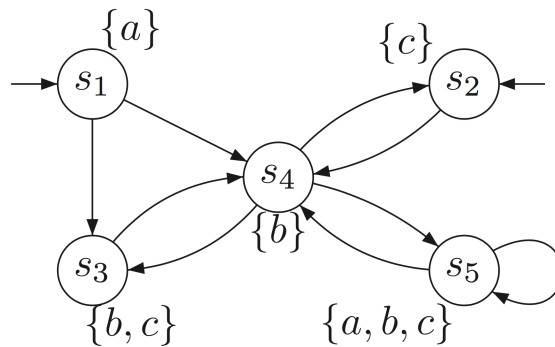


Figure 2.1:

Decide for each of the LTL formulae φ_i below, whether $TS \models \varphi_i$ holds. Justify your answers! If $TS \not\models \varphi_i$, provide a path $\pi \in \text{Paths}(TS)$ such that $\pi \not\models \varphi_i$

$$\begin{aligned}\varphi_1 &= \diamond \square c \\ \varphi_2 &= \square \diamond c \\ \varphi_3 &= \bigcirc \neg c \rightarrow \bigcirc \bigcirc c \\ \varphi_4 &= \square a \\ \varphi_5 &= a \cup \square (b \vee c) \\ \varphi_6 &= (\bigcirc \bigcirc b) \cup (b \vee c)\end{aligned}$$

3 PROBLEM 3

(20 points)

Suppose we have two users, *Peter* and *Betsy*, and a single printer device *Printer*. Both users perform several tasks, and every now and then they want to print their results on the *Printer*. Since there is only a single printer, only one user can print a job at a time. Suppose we have the following atomic propositions for *Peter* at our disposal:

- *Peter.request* ::= indicates that *Peter* requests usage of the printer;
- *Peter.use* ::= indicates that *Peter* uses the printer;
- *Peter.release* ::= indicates that *Peter* releases the printer.

For *Betsy*, similar predicates (propositions) are defined. Specify in LTL the following properties:

- (a) Mutual exclusion, i.e., only one user at a time can use the printer.
- (b) Finite time of usage, i.e., a user can print only for a finite amount of time.
- (c) Absence of individual starvation, i.e., if a user wants to print something, he/she eventually is able to do so.
- (d) Absence of blocking, i.e., a user can always request to use the printer
- (e) Alternating access, i.e., users must strictly alternate in printing.

4 PROBLEM 4

(20 points)

Consider an elevator system that services $N > 0$ floors numbered 0 through $N - 1$. There is an elevator door at each floor with a call-button and an indicator light that signals whether or not the elevator has been called. For simplicity consider $N = 4$. Present a set of atomic propositions - try to minimize the number of propositions - that are needed to describe the following properties of the elevator system as LTL formulae and give the corresponding LTL formulae:

- (a) The doors are “safe”, i.e., a floor door is never open if the elevator is not present at the given floor.
- (b) A requested floor will be served sometime.
- (c) Again and again the elevator returns to floor 0.
- (d) When the top floor is requested, the elevator serves it immediately and does not stop on the way there.

5 PROBLEM 5

(30 points - 10 + 10 + 5 + 5)

Which of the following equivalences (*if and only if*) are correct? Prove the equivalence or provide a counterexample that illustrates that the formula on the left and the formula on the right are not equivalent.

- (a) $\Box\varphi \rightarrow \Diamond\psi \equiv \varphi \cup (\psi \vee \neg\varphi)$
- (b) $\Diamond\Box\varphi \rightarrow \Box\Diamond\psi \equiv \Box(\varphi \cup (\psi \vee \neg\varphi))$
- (c) $\Box\Box(\varphi \cup \neg\psi) \equiv \neg\Diamond(\neg\varphi \wedge \psi)$
- (d) $\Diamond(\varphi \wedge \psi) \equiv \Diamond\varphi \wedge \Diamond\psi$