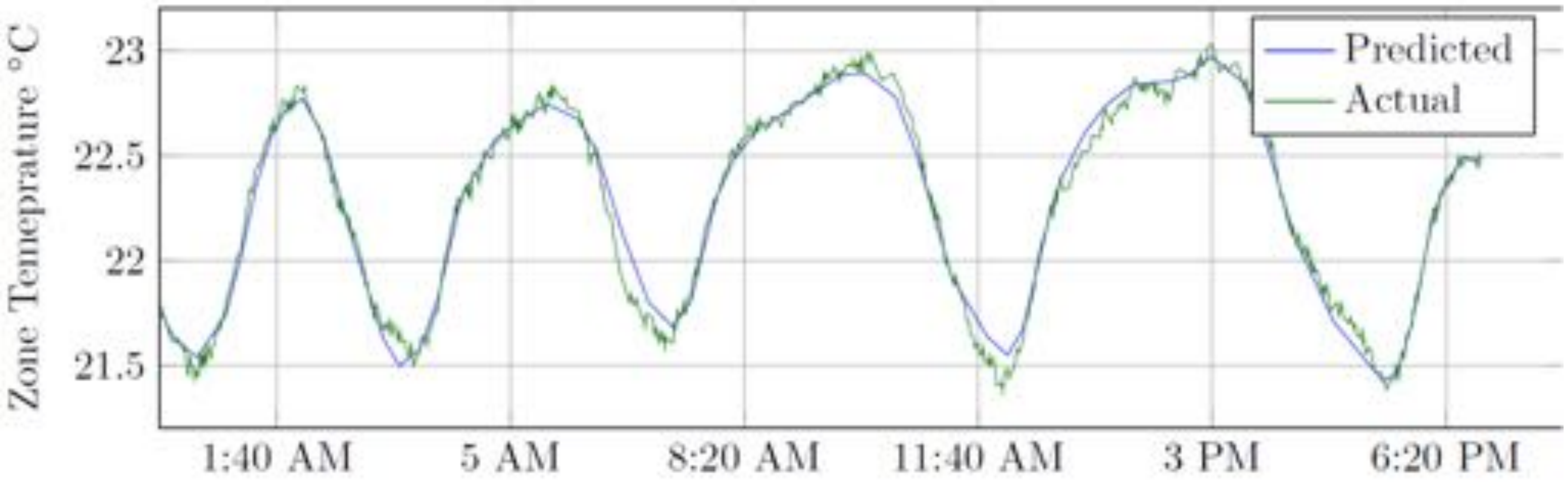
Model Sensitivity Analysis

Lecture 8

Principles of Modeling for Cyber-Physical Systems

Instructor: Madhur Behl

How do I know my model is any good ?



Sensitivity Analysis

In general $y = f(\theta, x(t), u(t))$

We want to attribute, the uncertainty in **y** to the uncertainty and errors in parameters $\boldsymbol{\theta}$, and inputs **u**

Sensitivity = $\dfrac{\textit{How much does the output y change}}{\textit{for a change in a single parameter or input}}$ subject to, all other things being the same.

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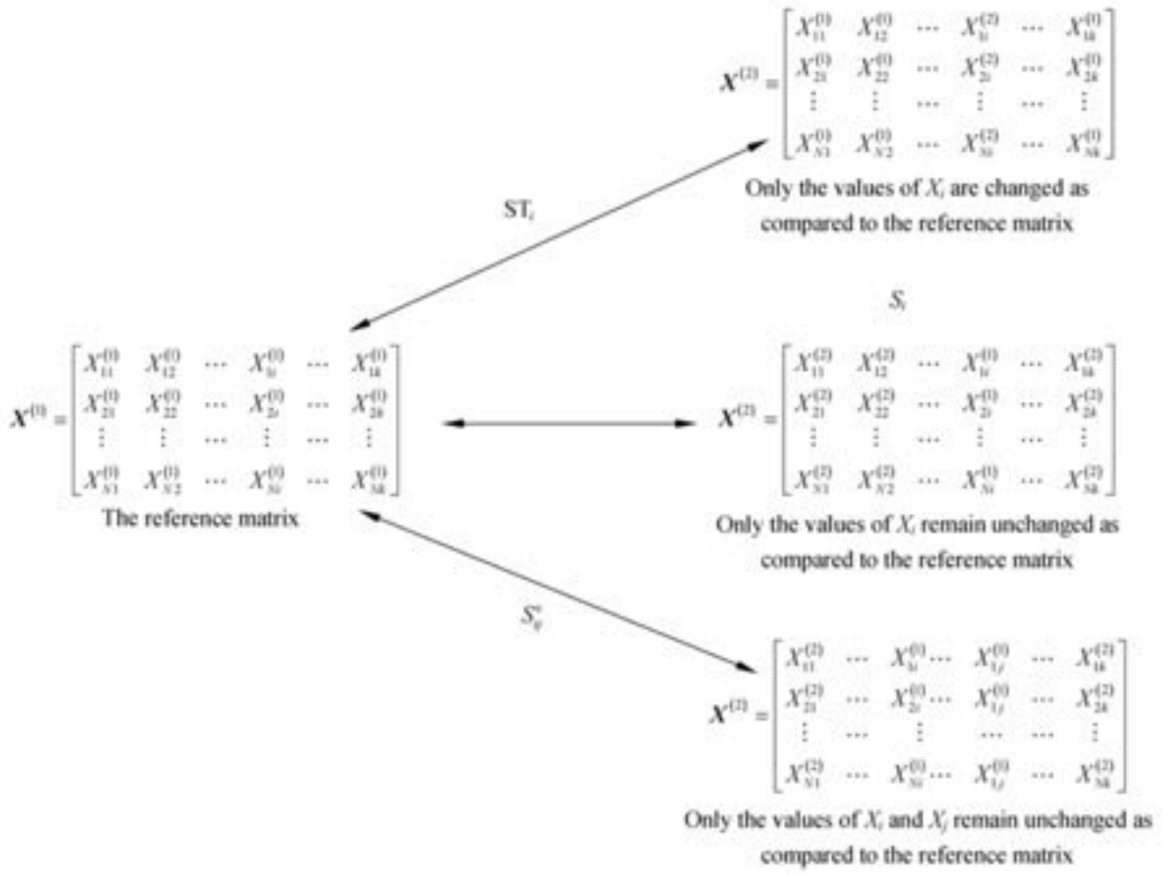
Sensitivity Analysis



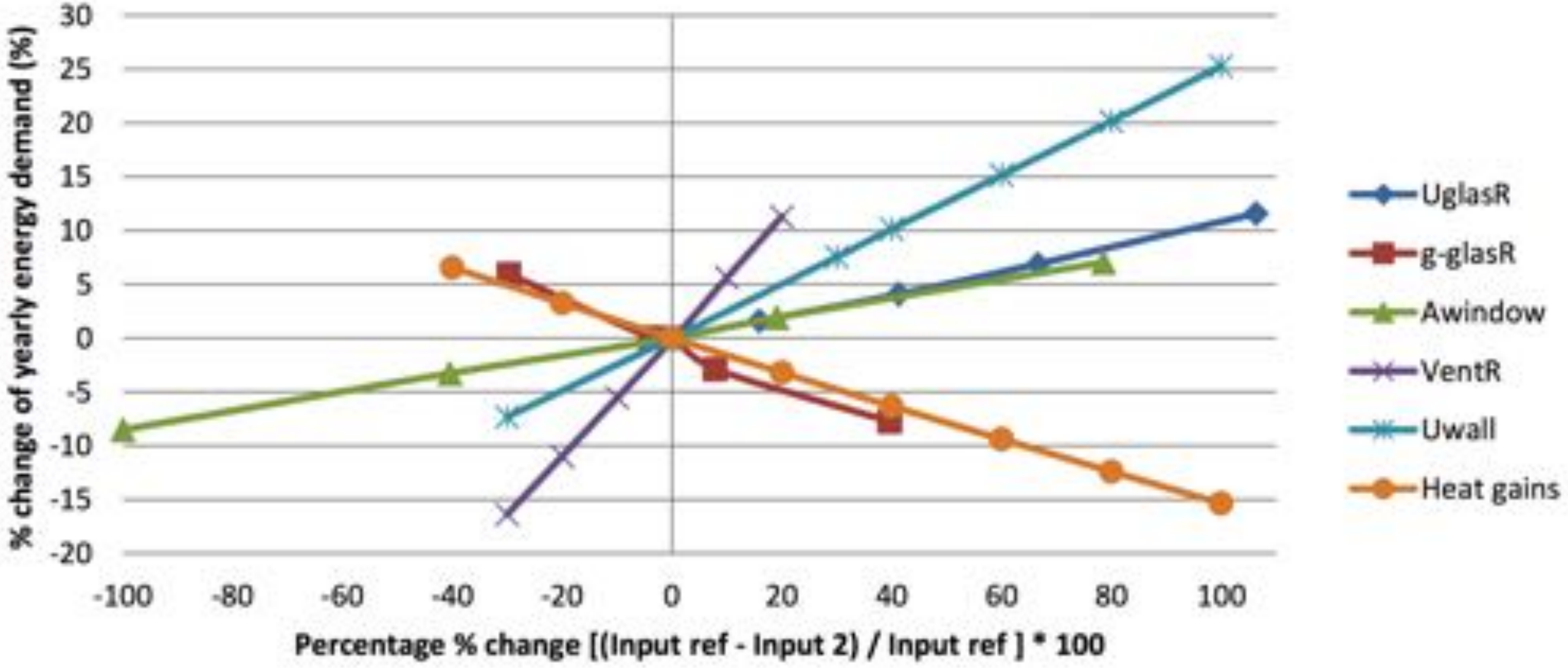
Input-Output Sensitivity

$$\frac{\Delta y}{\Delta u}$$

Parameter Sensitivity

$$\frac{\Delta y}{\Delta \theta}$$

Sensitivity Analysis



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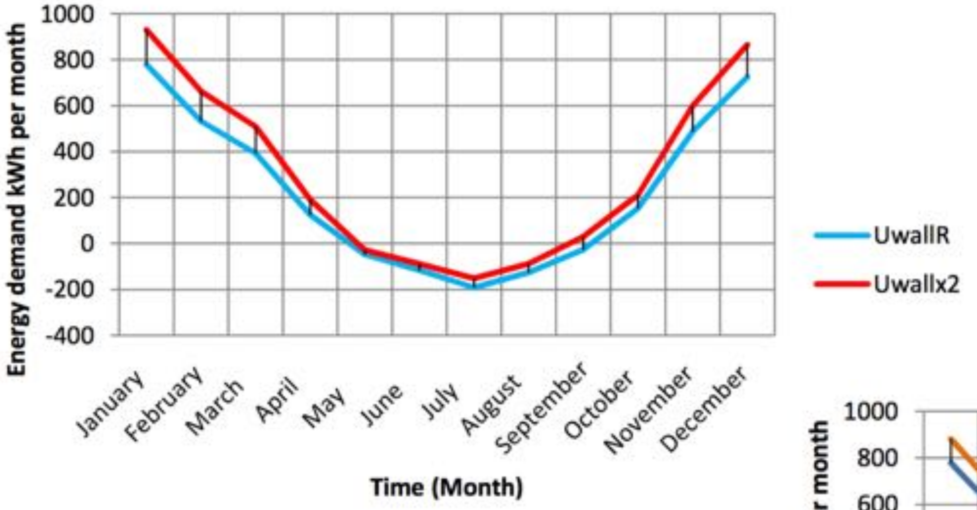
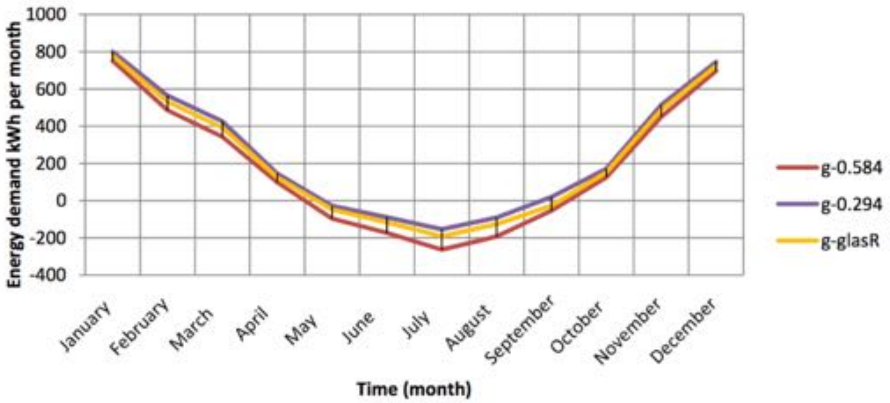
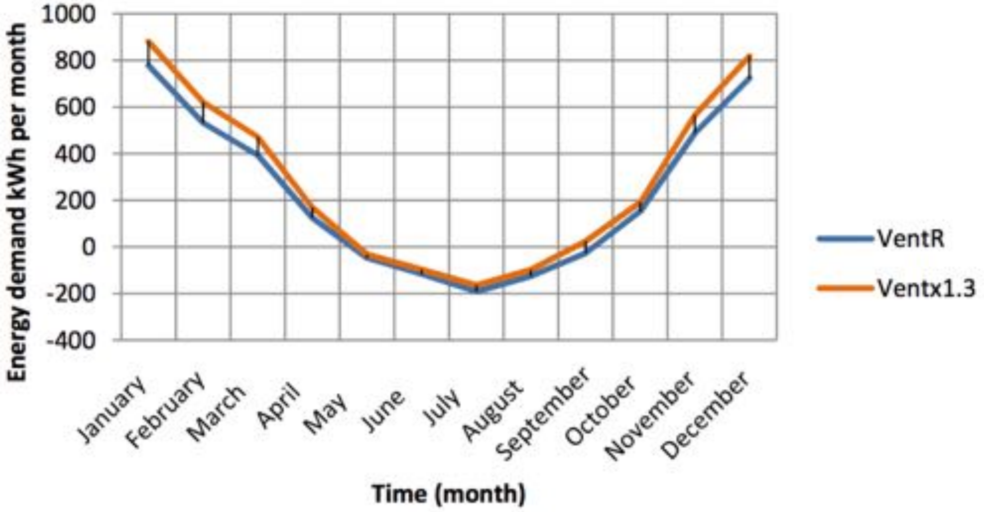
Sensitivity Analysis

Abbreviation	Input parameter	Influence coefficient(2-3) (% OP per % IP)
U_{wall}	wall heat transfer	0.206
U_{glass}	glass heat transfer	0.098
g_{glass}	solar gain glass	-0.211
Heat gains	Amount of heat gain	-0.198
Awindow	window frame-to-glass ratio	0.123
VentR	Ventilation rate	0.485

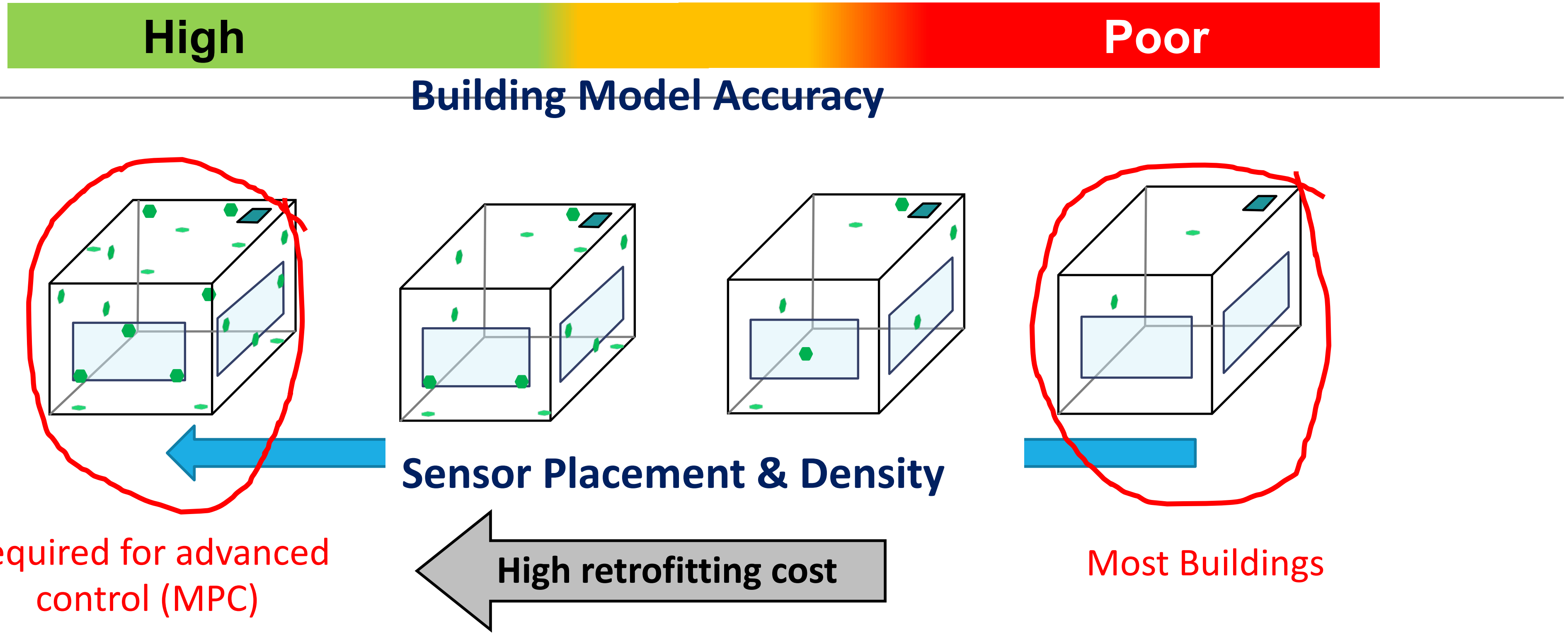
Sensitivity Analysis







Better models for better control..



Small and medium sized commercial buildings (**90% of the commercial building stock**) do not want to spend thousands of dollars on retrofitting.

“Accuracy costs money,
how accurate do you want it ?”

Sensor Data Quality vs Building Model Accuracy?

Two thermostats/actuators, same objective

Sensor Data Quality and Uncertainty

1) Due to Sensor Placement and Density

Image courtesy Bryan Eisenhower (IMA talk)

2) Due to Sensor Precision

vs

$500 **$1**

3) Due to Inference: E.g. Heat gains from Occupancy measured with people counters

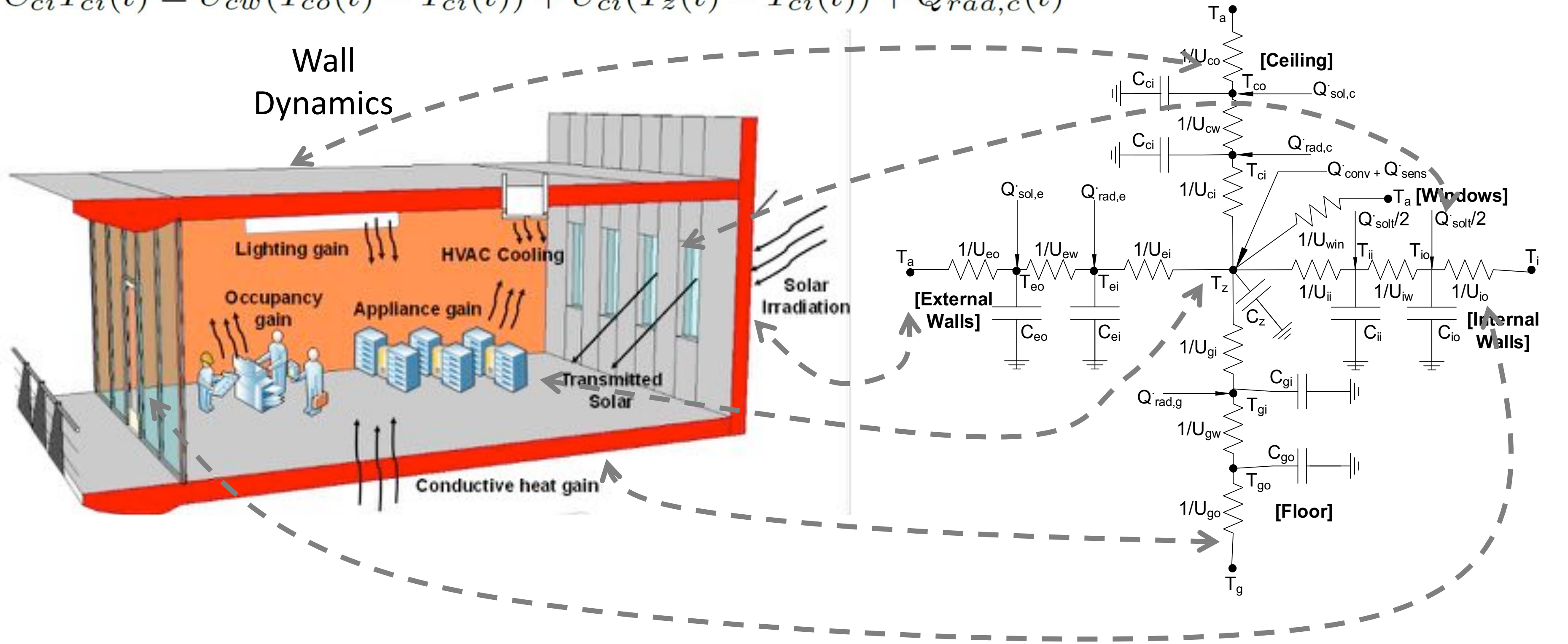
4) Measurement Noise

Building Modeling: "RC-Networks"

Measure all Inputs and Disturbances
Ambient temperature, convective heat gain, radiative heat gain, external solar gain, ground temperature, cooling rate

$$C_{co}\dot{T}_{co}(t) = U_{co}(T_a(t) - T_{co}(t)) + U_{cw}(T_{ci}(t) - T_{co}(t)) + \dot{Q}_{sol,c}(t)$$

$$C_{ci}\dot{T}_{ci}(t) = U_{cw}(T_{co}(t) - T_{ci}(t)) + U_{ci}(T_z(t) - T_{ci}(t)) + \dot{Q}_{rad,c}(t)$$



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Building Modeling: "RC-Networks"

Discrete-Time State Space Model:

(parameterized by θ)

$$x(k+1) = \hat{A}_\theta x(k) + \hat{B}_\theta u(k)$$
$$y(k) = \hat{C}_\theta x(k) + \hat{D}_\theta u(k)$$

States (**All node temperatures**):

$x = [T_{eo}, T_{ei}, T_{co}, T_{ci}, T_{go}, T_{gi}, T_{io}, T_{ii}, T_z]^T$

Inputs (**Disturbances and Control**):

$u = [T_a, T_g, T_i, Q_{sole}, Q_{solc}, Q_{rade}, Q_{radc}, Q_{radg}, Q_{solt}, Q_{conv}, \mathbf{Q_{sens}}]^T$

Parameter Estimation:

Least Squares Error

$$\theta^* = \underset{\theta_l \leq \theta \leq \theta_u}{\arg\min} \sum_{k=1}^{N} (T_{z_m}(k) - T_{z_\theta}(k))^2$$

$$\text{subject to } \theta_l \leq \theta \leq \theta_u$$

LIST OF PARAMETERS

$U_{\star o}$	convection coefficient between the wall and outside air
$U_{\star w}$	conduction coefficient of the wall
$U_{\star i}$	convection coefficient between the wall and zone air
U_{win}	conduction coefficient of the window
$C_{\star\star}$	thermal capacitance of the wall
C_z	thermal capacity of zone z_i

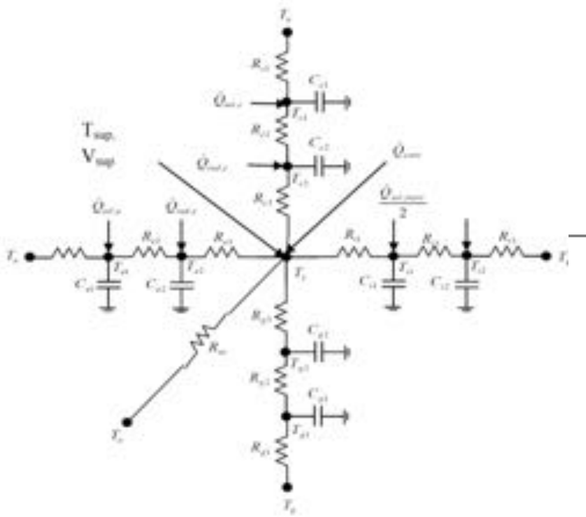
g: floor; e: external wall; c: ceiling; i: internal wall

Accuracy of an Inverse Model

1) Model Structure

FIXED

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$



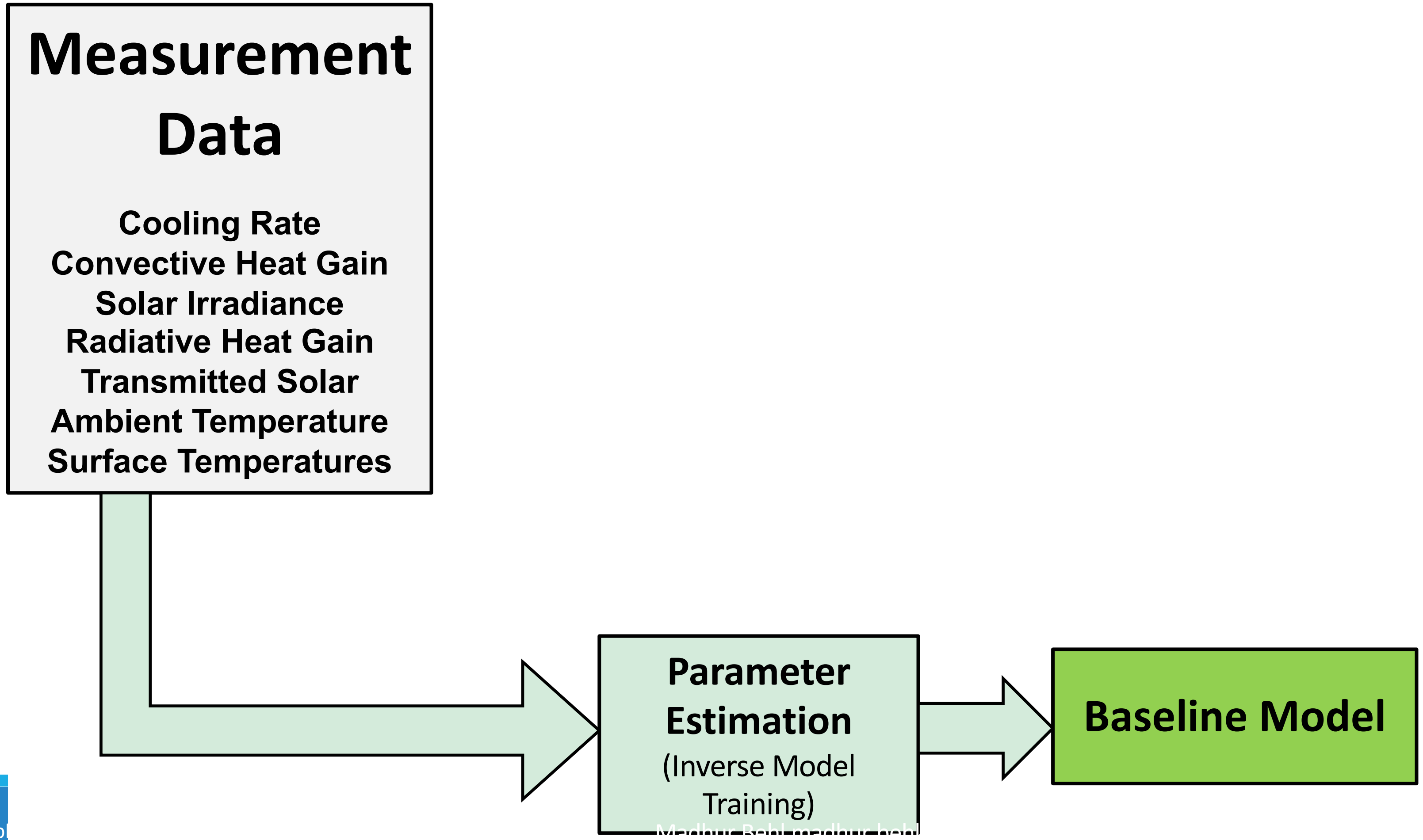
2) Parameter estimation algorithm

FIXED

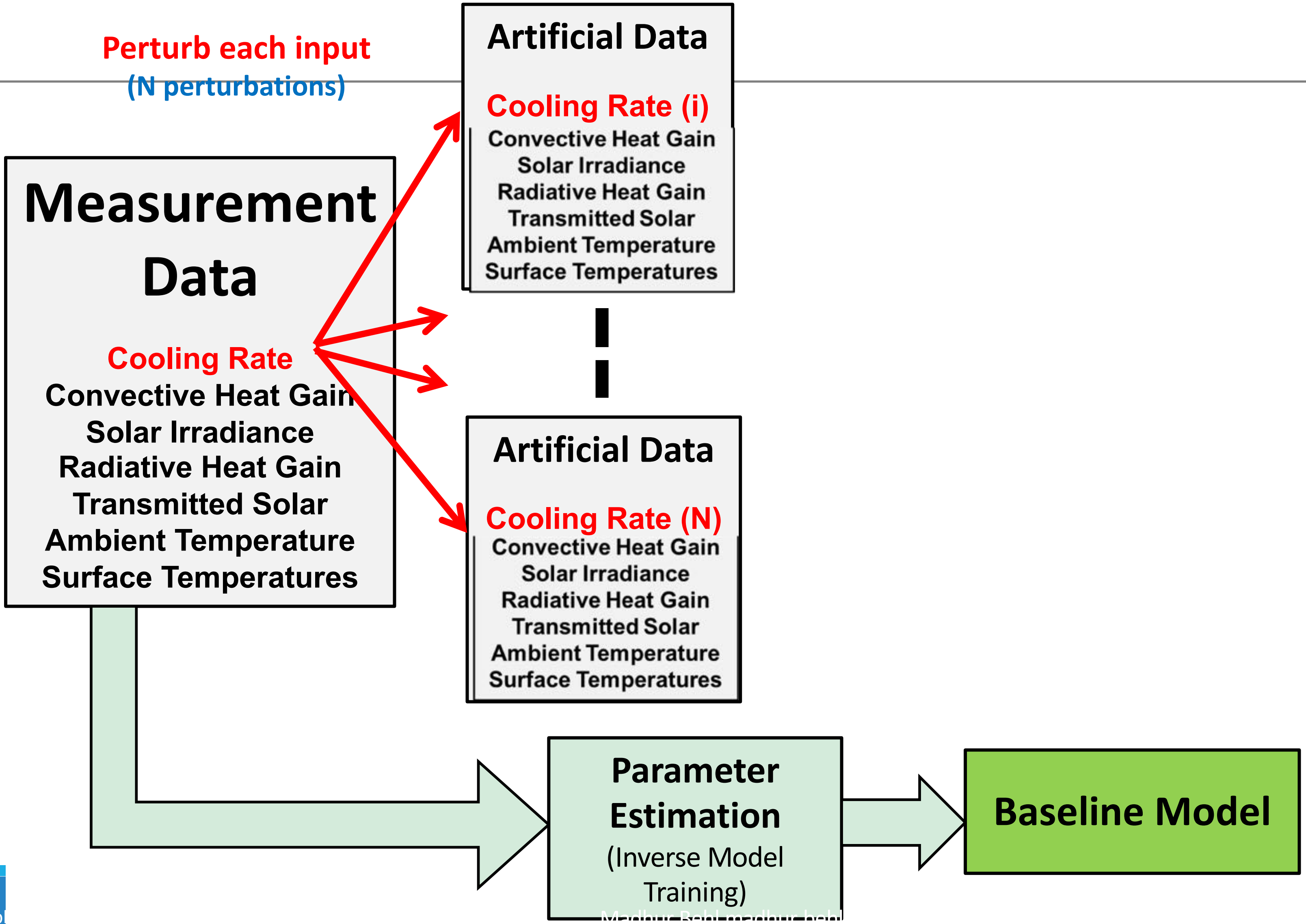
Non-Linear regression

3) Uncertainty in the input-output data

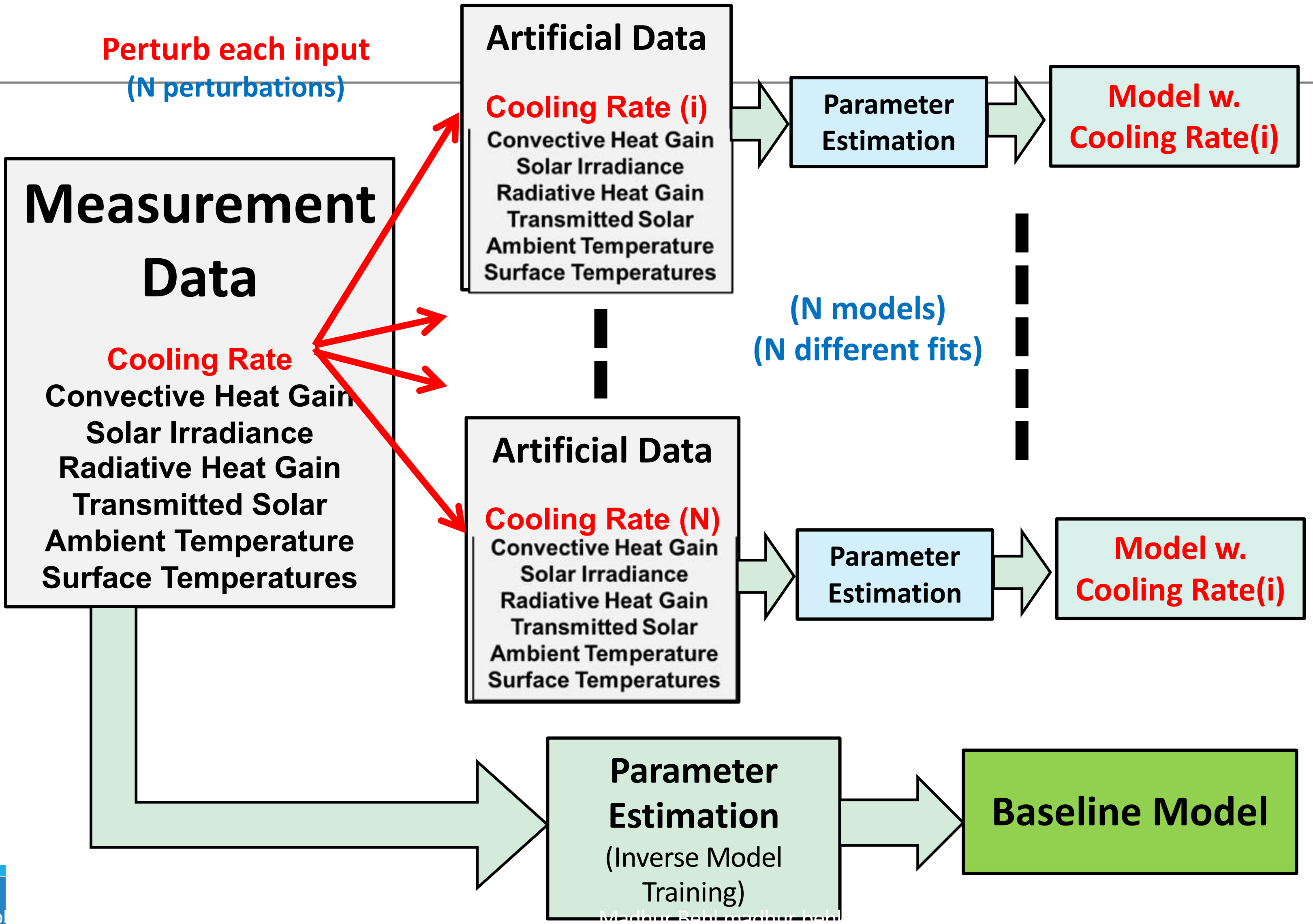
Input Uncertainty Analysis



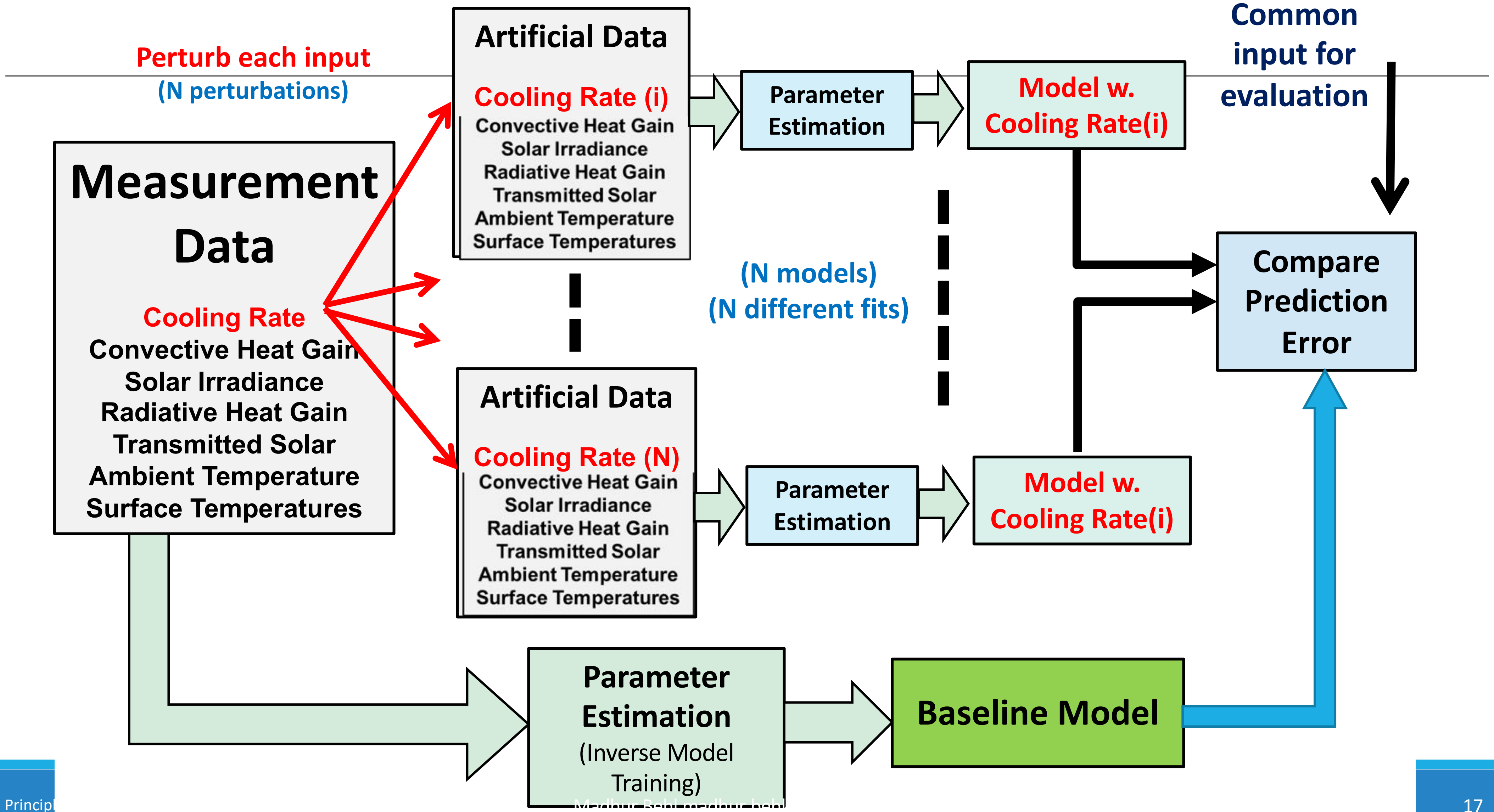
Input Uncertainty Analysis



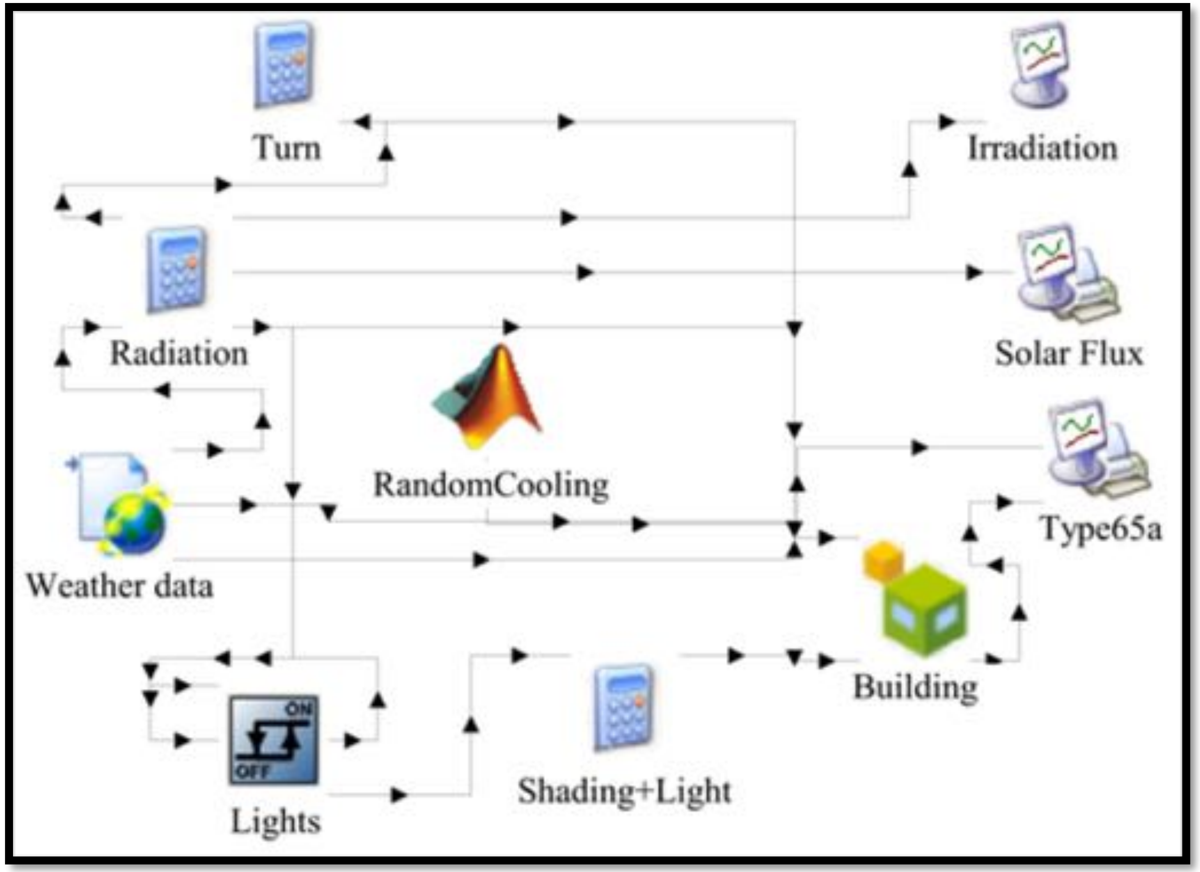
Input Uncertainty Analysis



Input Uncertainty Analysis

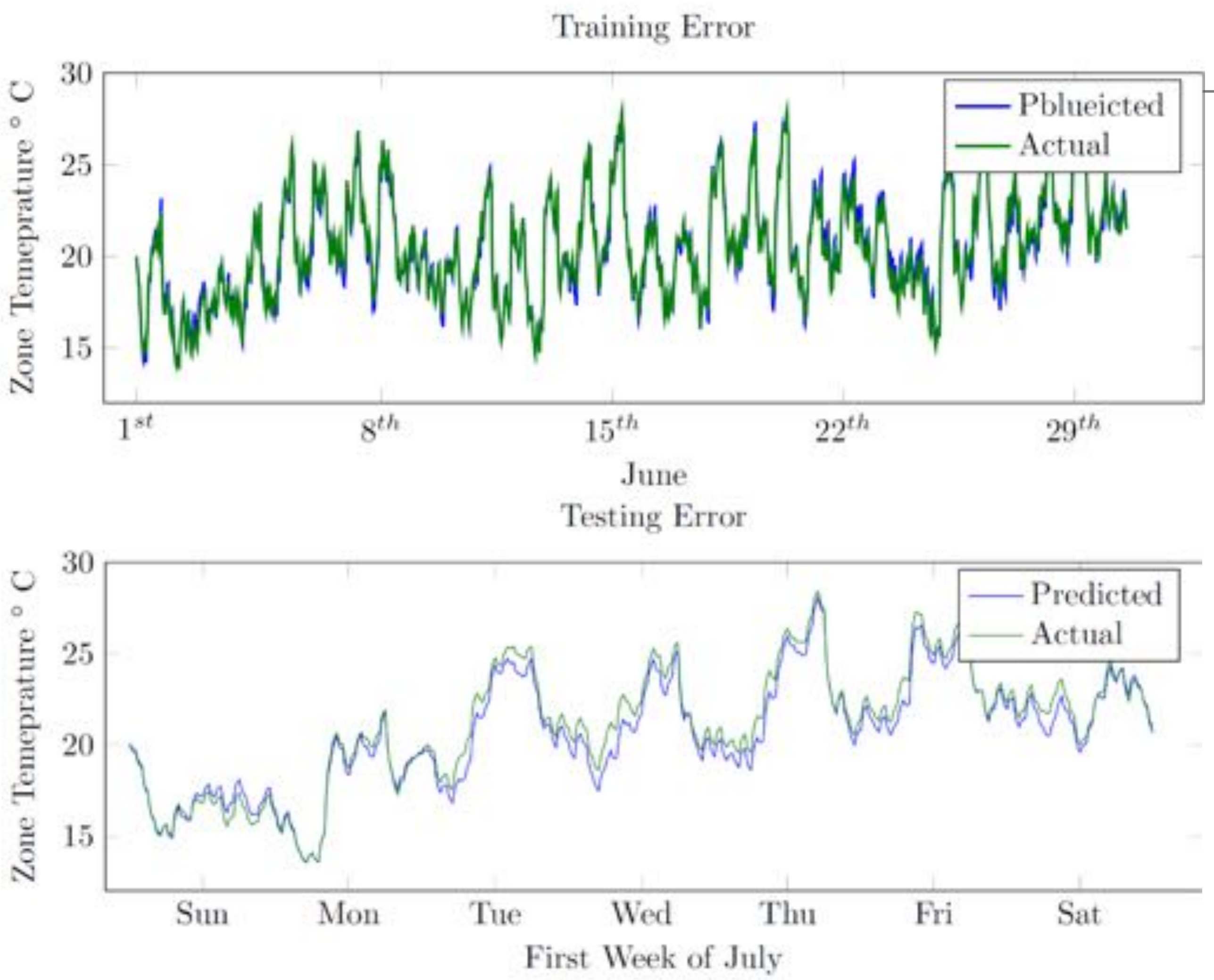


Uncertainty analysis with TRNSYS building



- North Facing
- 4 external brick walls
- 4 large windows
- Concrete floor and ceiling
- Philadelphia-TMY2 weather
- 3.5kW HVAC system

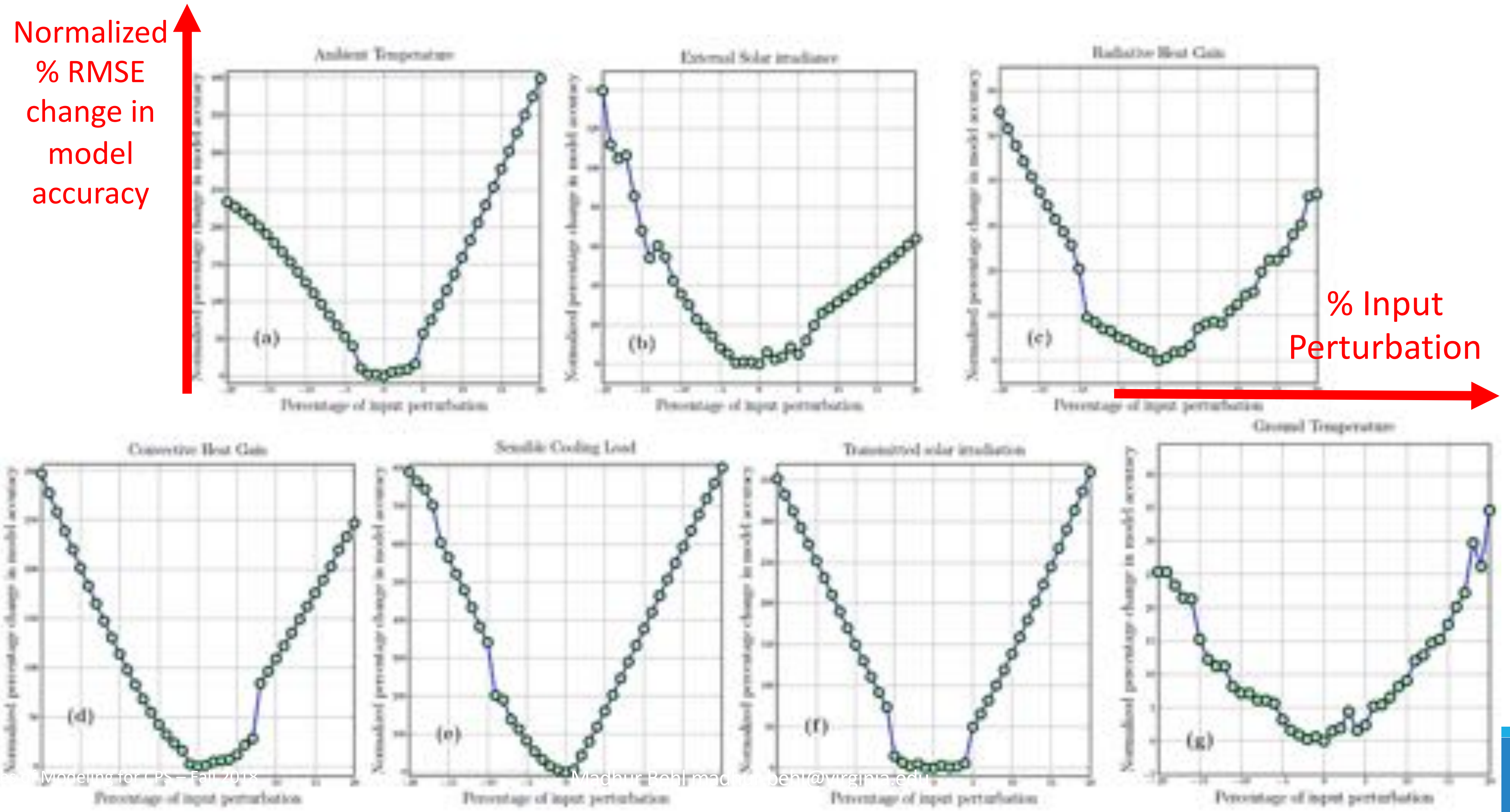
Uncertainty analysis with TRNSYS building



- 12 RC parameters
- 7 inputs, 1 output
- **Baseline Model: RMSE 0.187 °C, R^2 0.971**
- Introduce fixed perturbations/bias in each input: $z_i' = z_i \pm (\delta * z_i)$

Uncertainty analysis with TRNSYS building

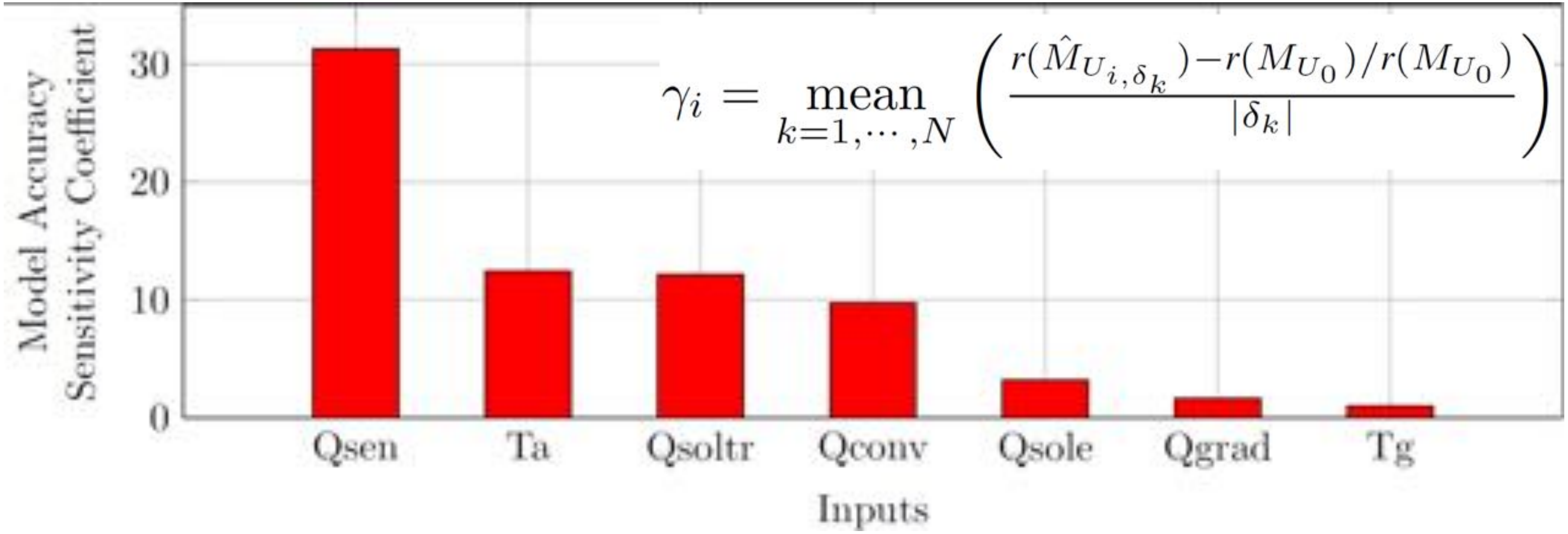
Ambient temperature, convective heat gain, radiative heat gain, external solar gain, ground temperature, sensible cooling load



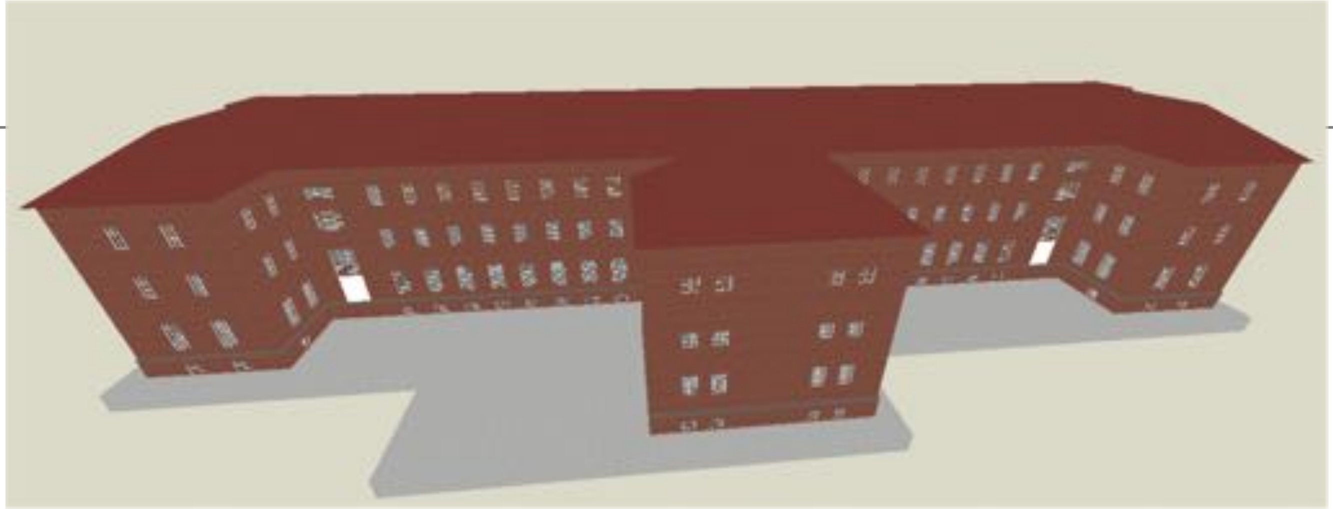
Uncertainty analysis with TRNSYS building

Model Accuracy Sensitivity Coefficient
(for input u)

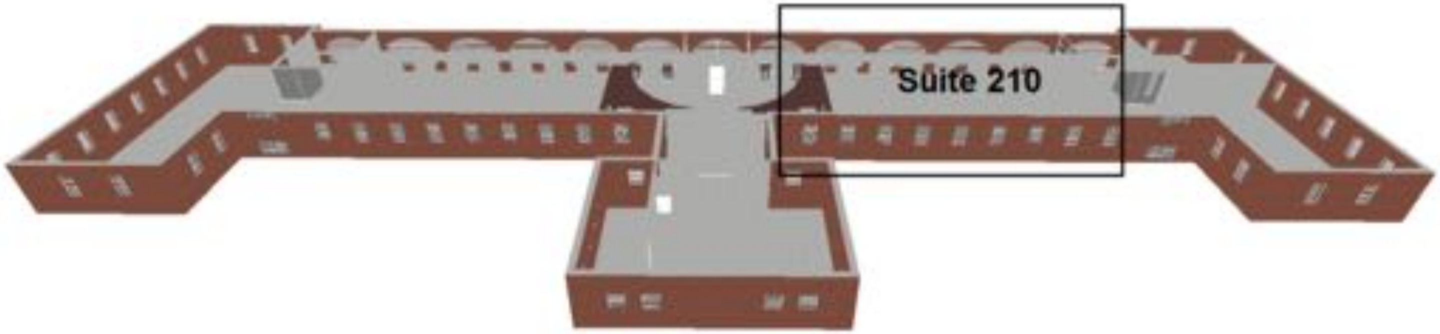
$$mean\left(\frac{Normalized\ change\ in\ model\ accuracy}{Normalized\ input\ perturbation}\right)$$



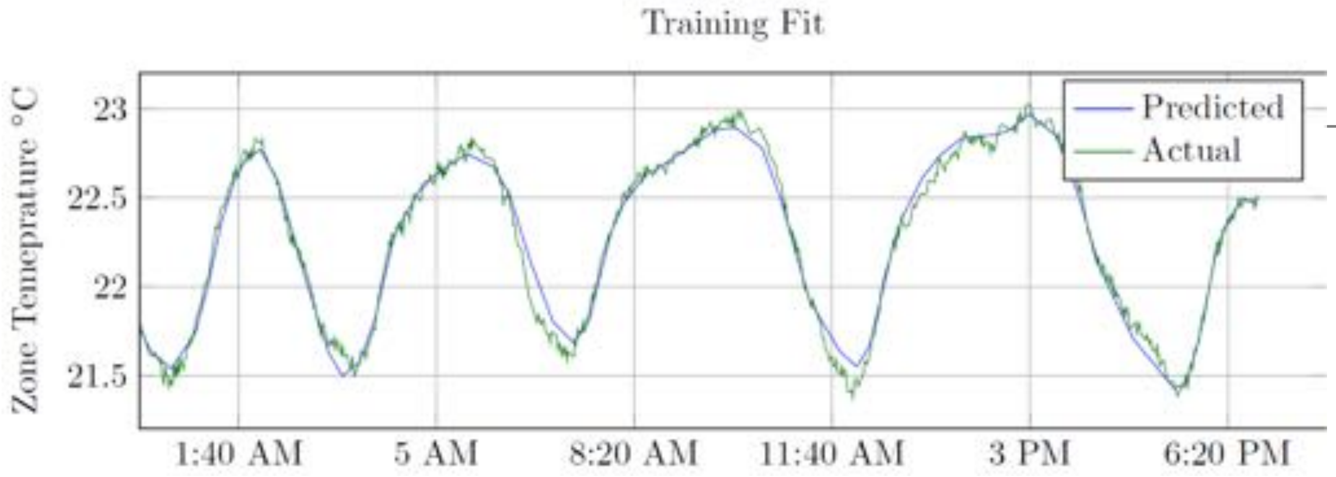
Case study: Building 101



Building 101 is located in Philadelphia
and it's the US DoE's Energy Efficient Buildings Hub Headquarter



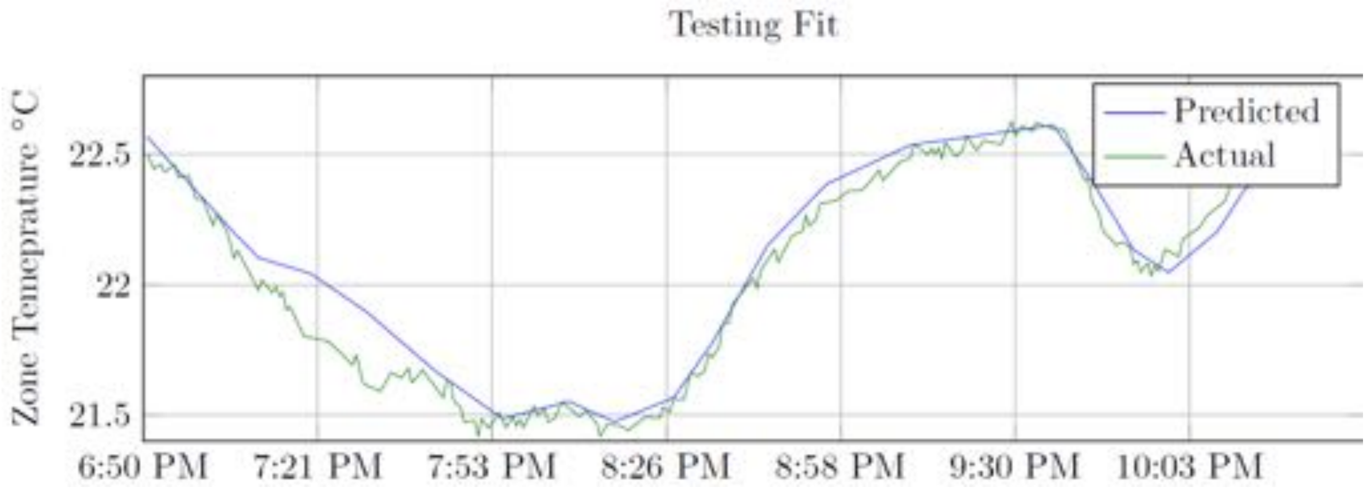
Case study: Building 101



Model Accuracy for Training data

RMSE: 0.062 °C
R2: 0.983

Baseline

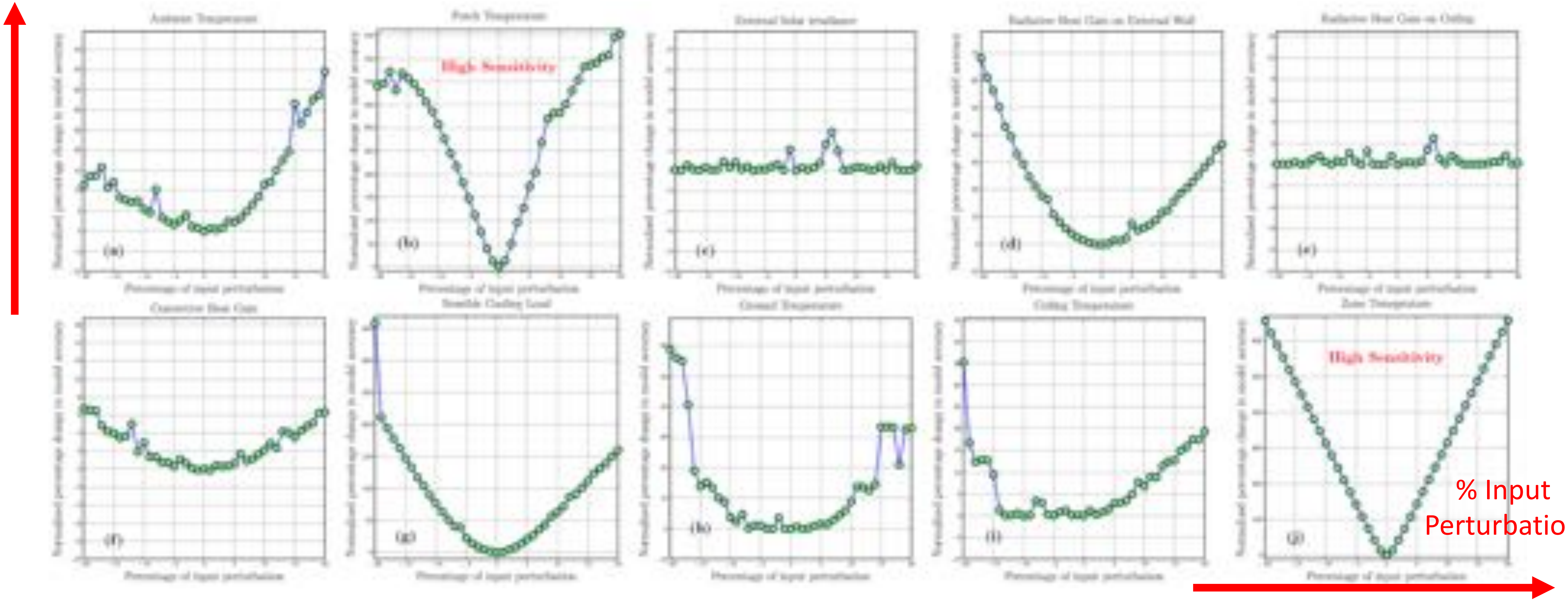


Model Accuracy for Test Data

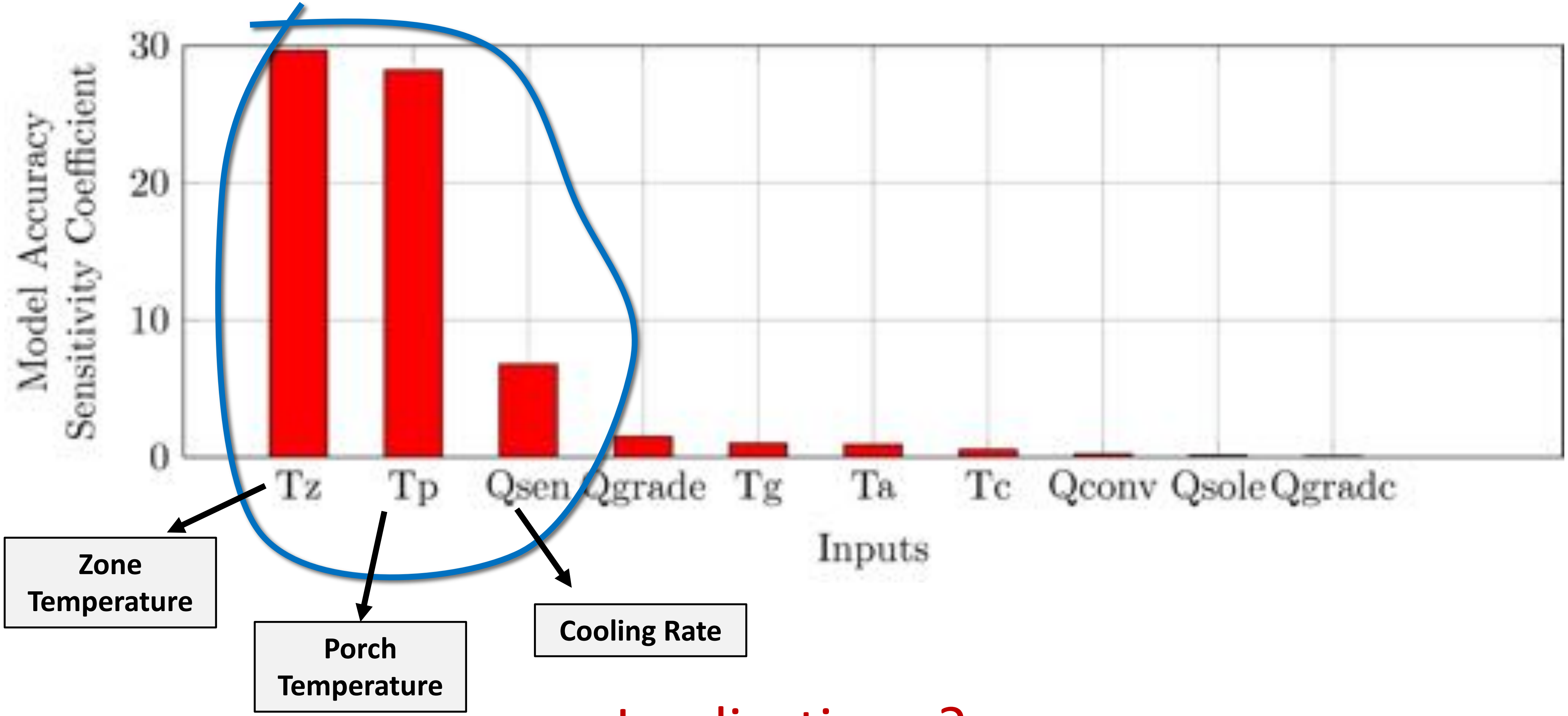
RMSE: 0.091 °C
R2: 0.948

Input Uncertainty Analysis: Building 101

Normalized % RMSE change in model accuracy



Model Accuracy Sensitivity Coefficient: Building 101



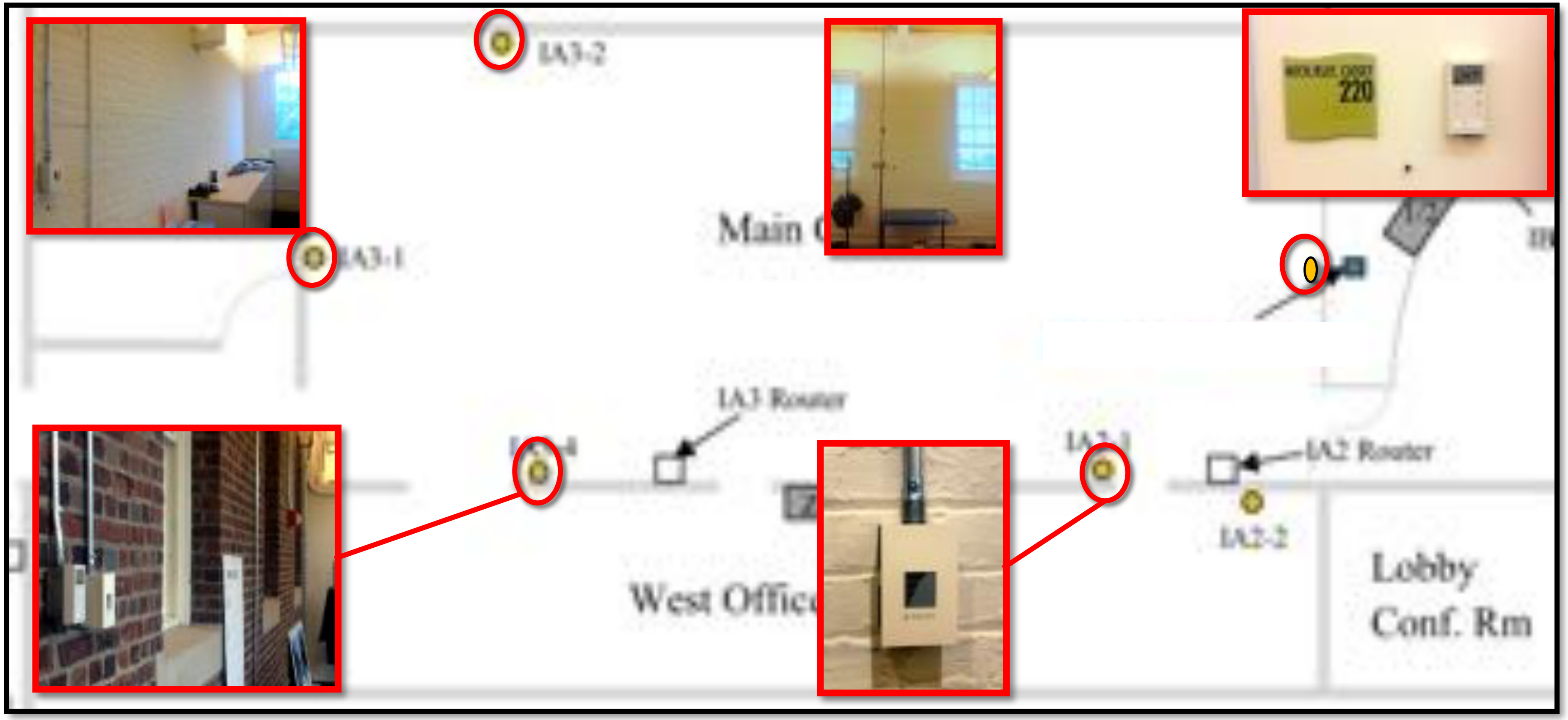
Implications ?

Sensor Placement and Quality of Data: Suite 210

4 Indoor Air Quality Sensors

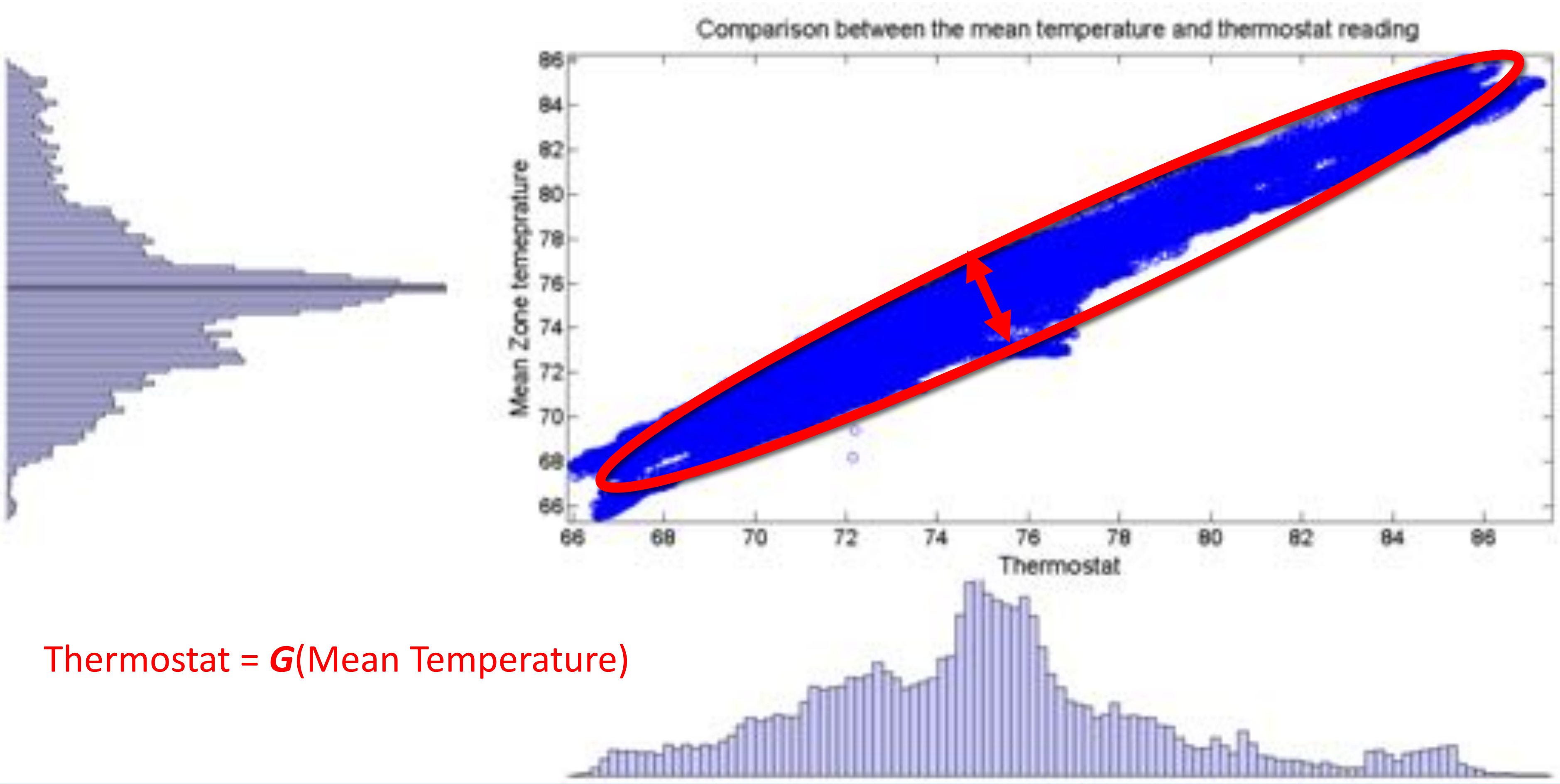
1 portable Cart

Zone Thermostat



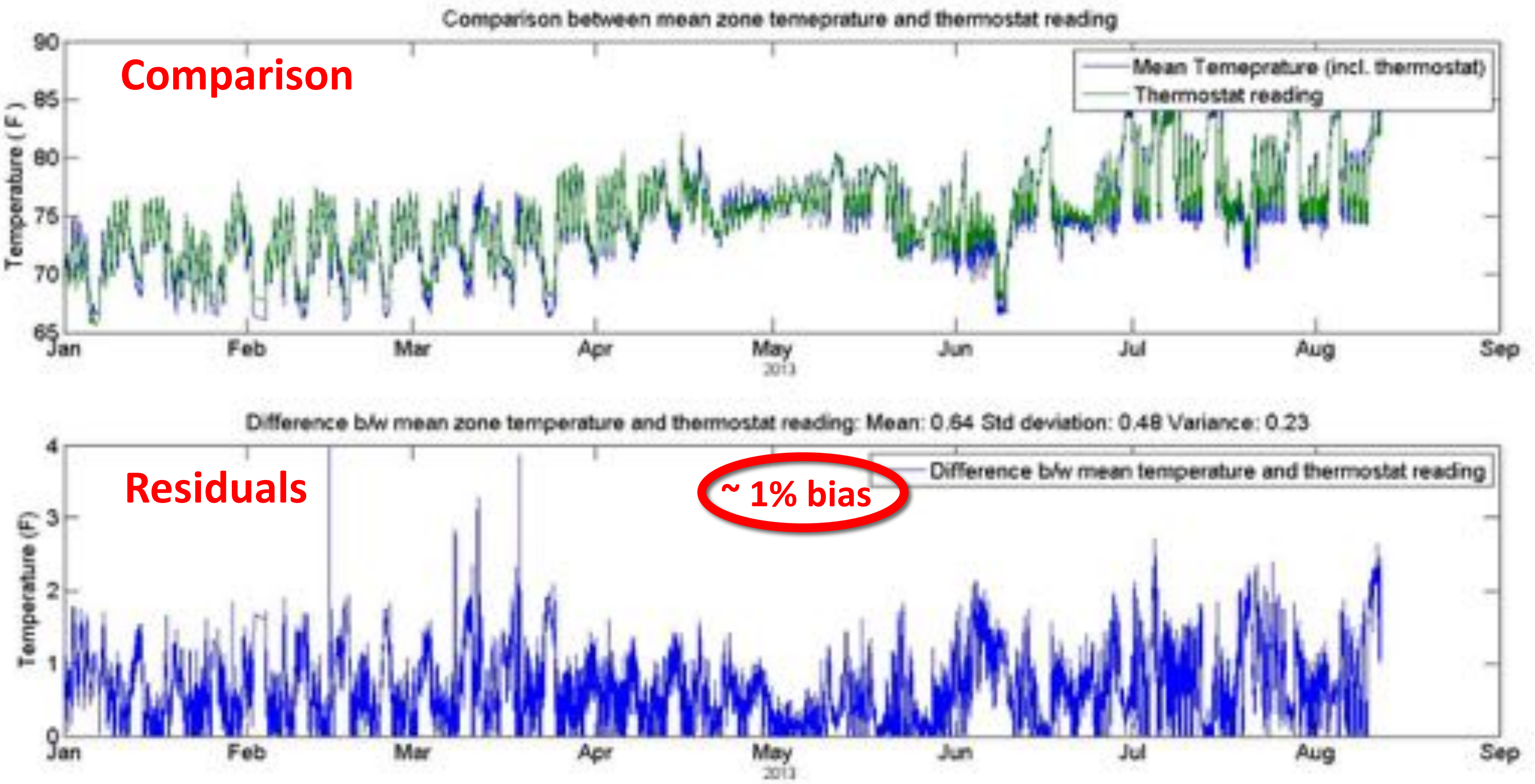
Is there a bias in the Thermostat data due to its location ?

Compare the “true” (mean) temperature with thermostat measurement

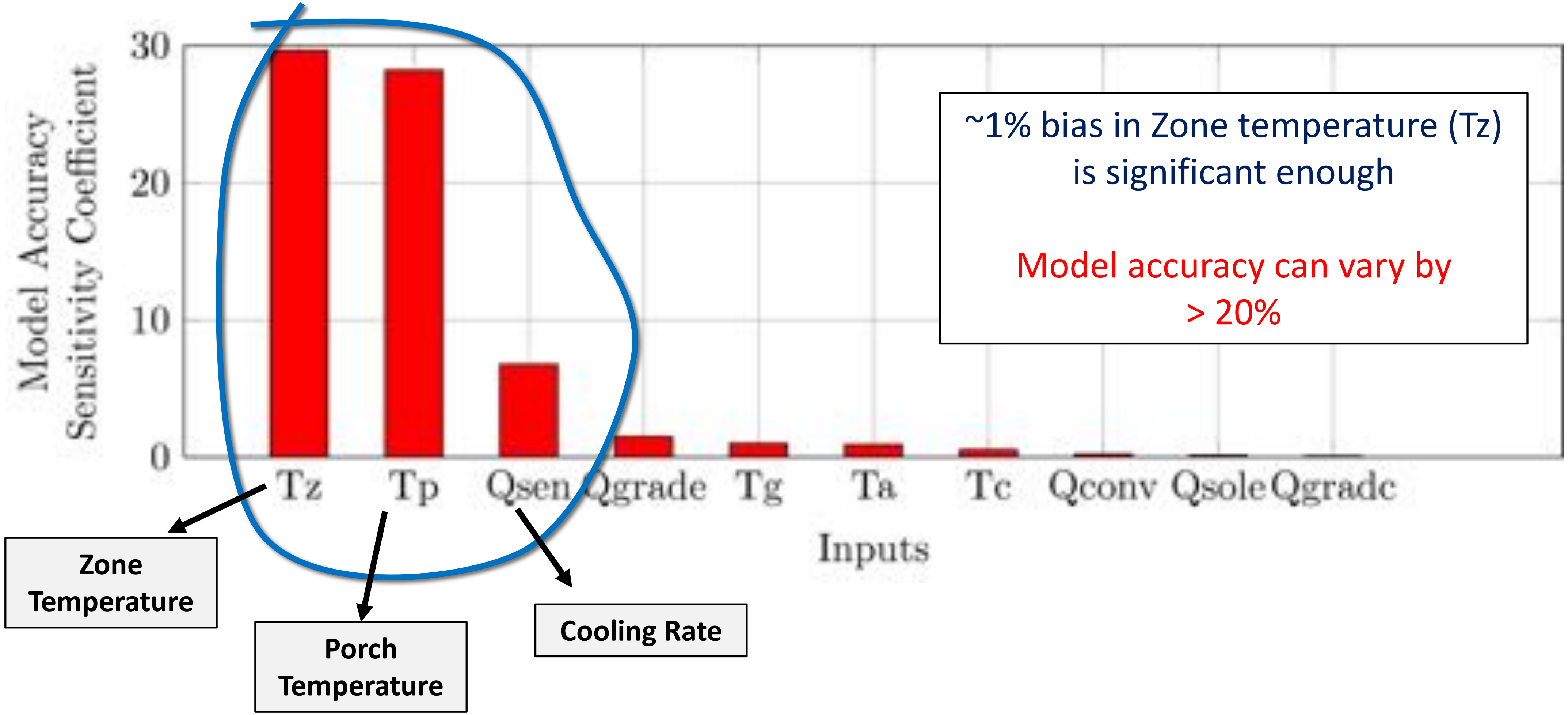


Thermostat = ***G***(Mean Temperature)

Sensor Placement and Quality of Data: Suite 210



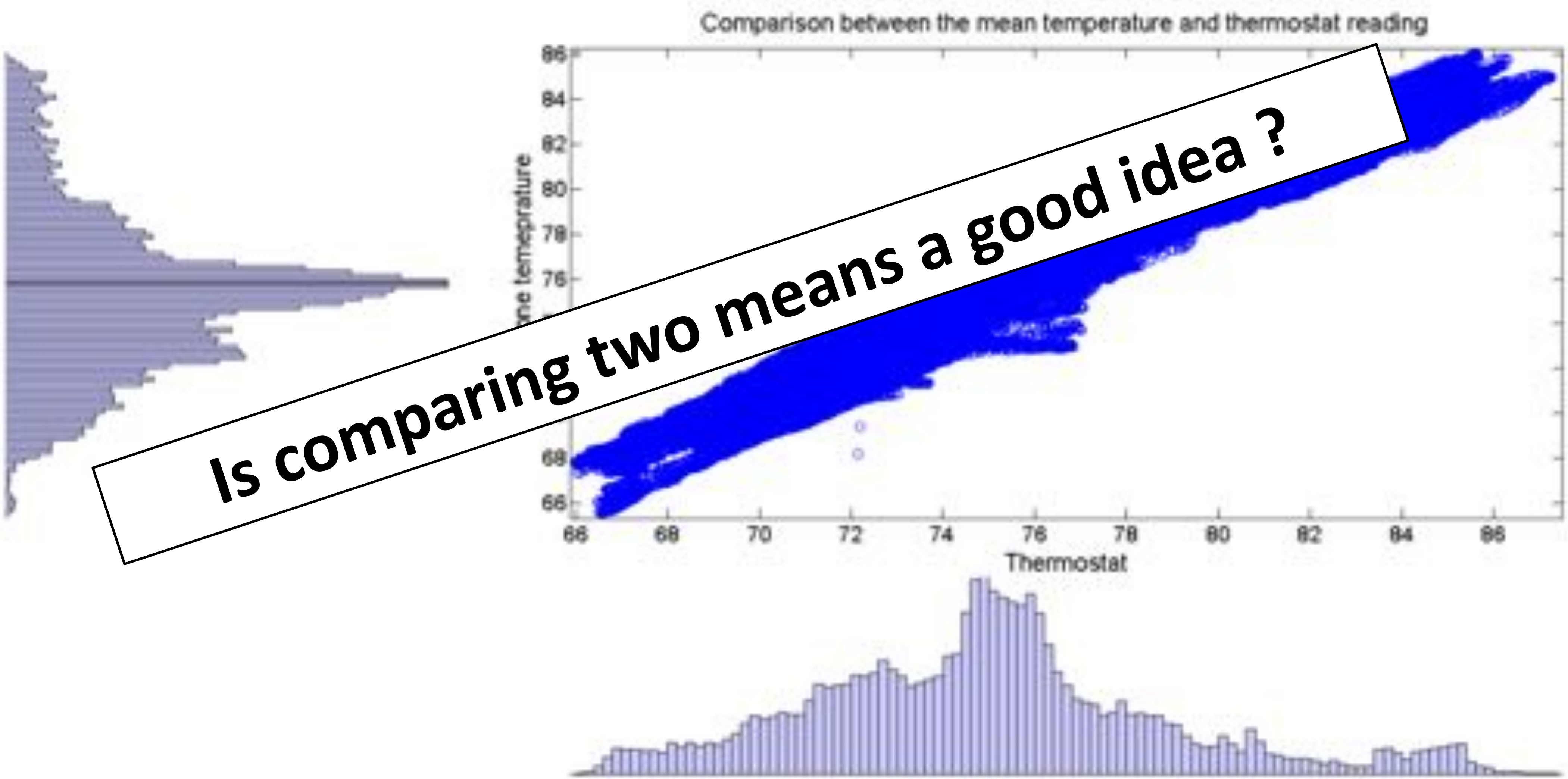
Sensor Placement and Quality of Data: Suite 210



~1% bias in Zone temperature (Tz) is significant enough

Model accuracy can vary by > 20%

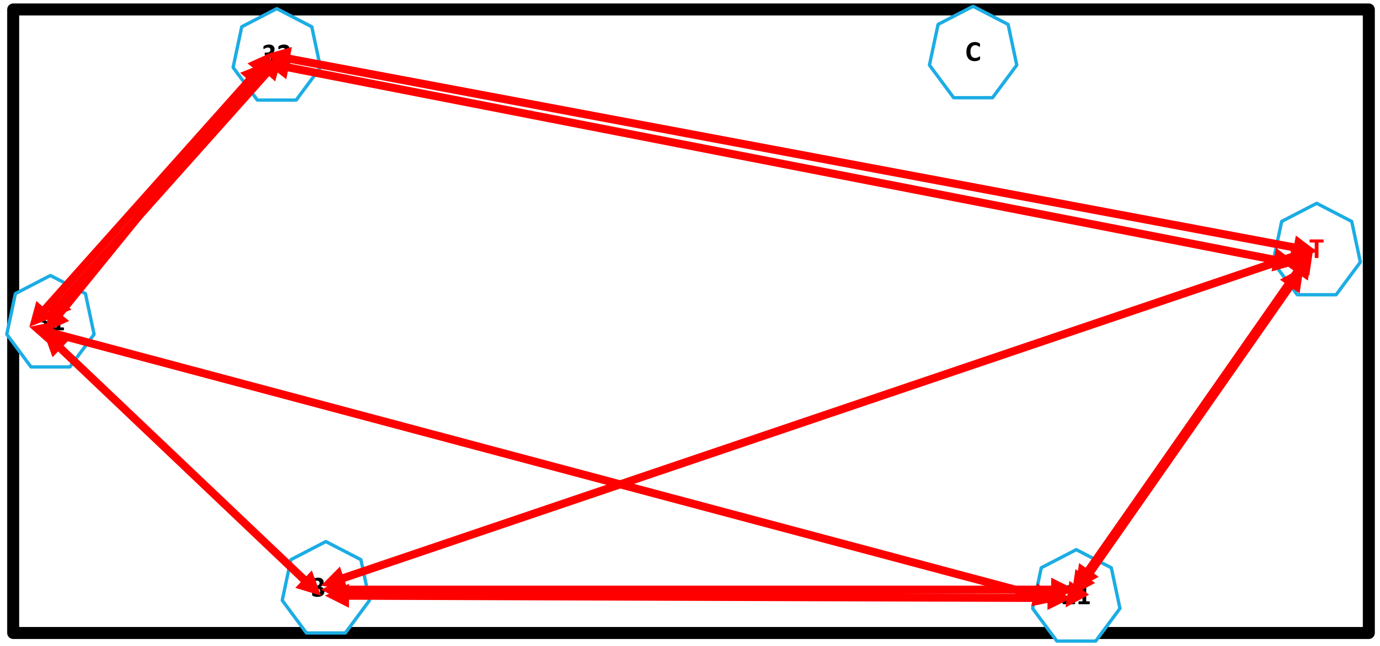
Sensor Placement and Bias



Is comparing two means a good idea ?

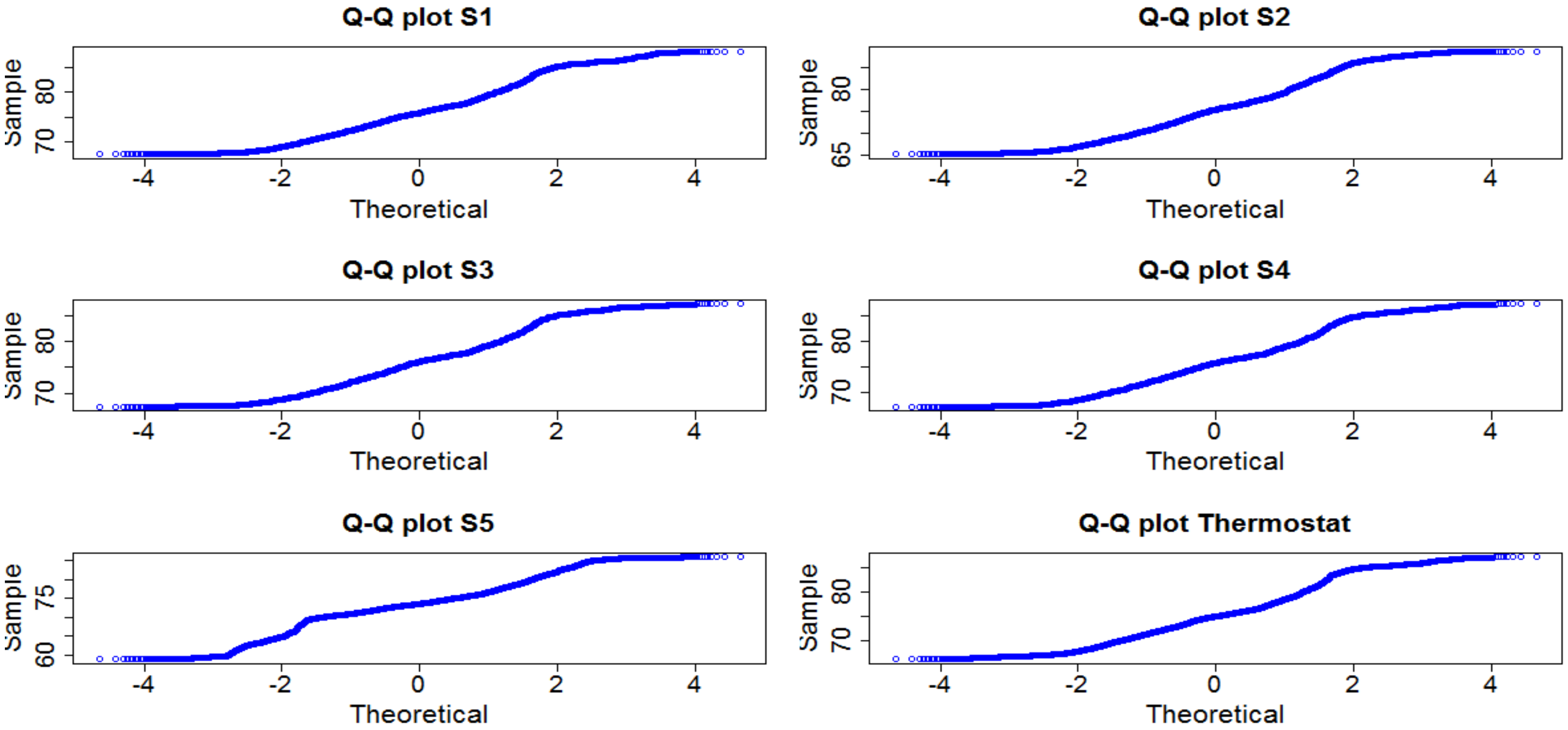
Maybe not..

Multiple subsets could be compared



A closer look at temperature data

Temperature sensor data is not normal (Gaussian)



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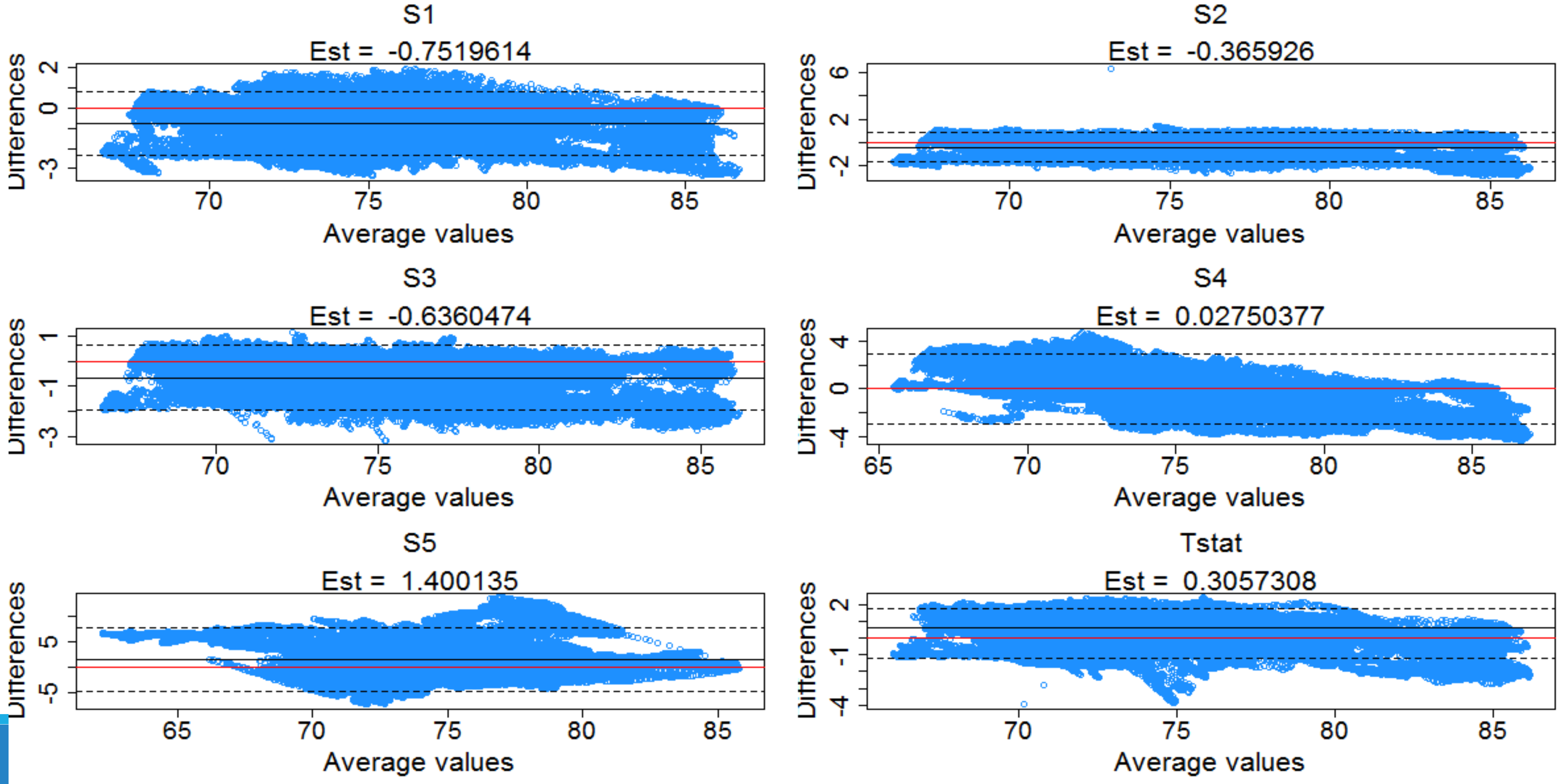
Non-parametric statistical methods

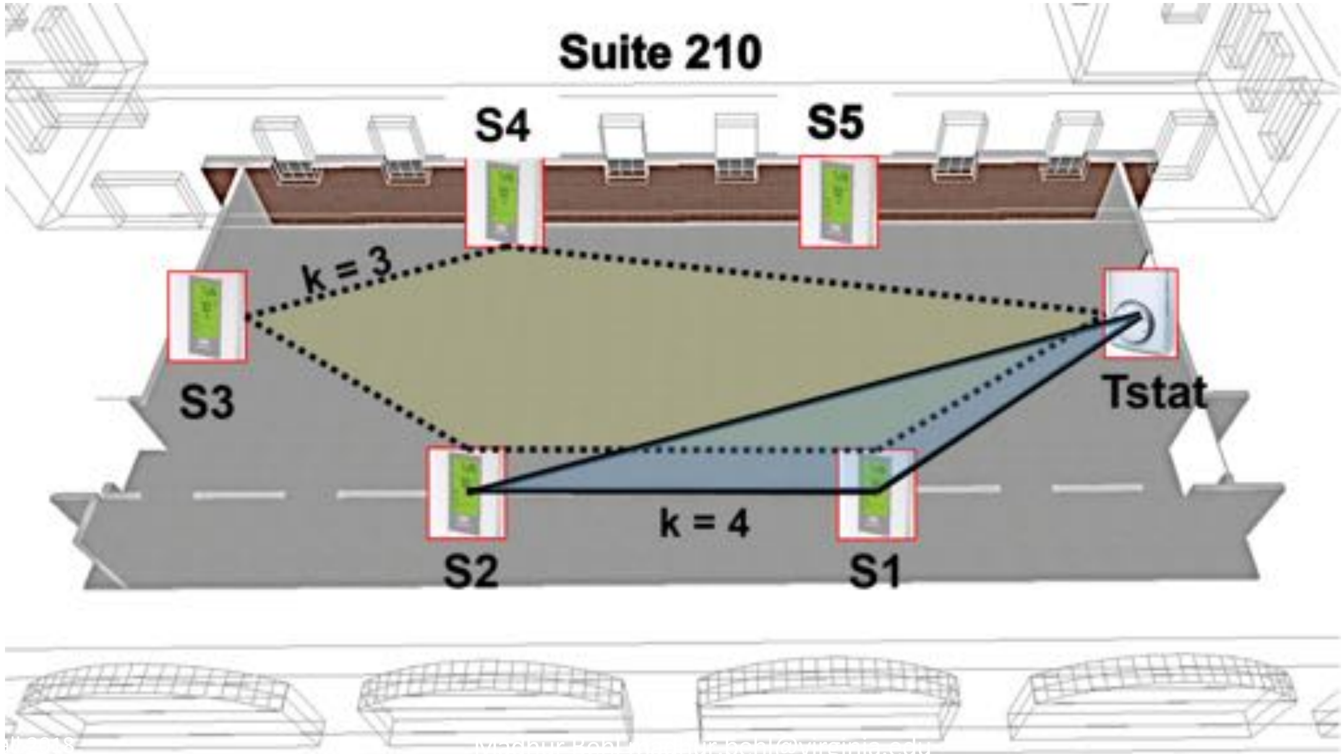
Use **Wilcoxon's rank sum test** and **Bland-Altman** plots to quantify bias and identify best sensor placements.



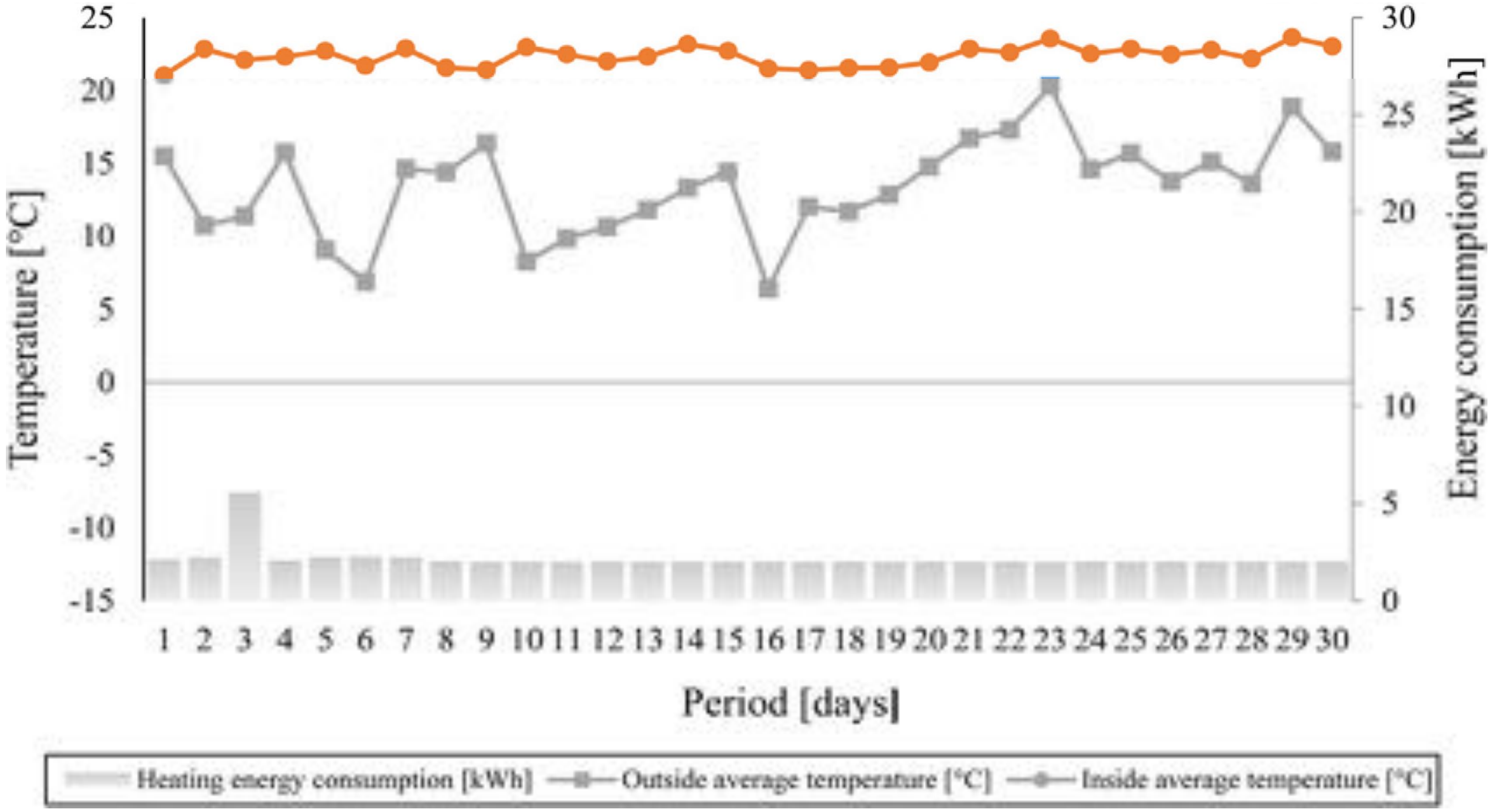
Non-parametric statistical methods

TABLE II: Wilcoxon's test results for all values of k

k	Min. bias subset T_k	Bias Estimate μ_k
1	s_4	0.0275
2	s_3, s_4	−0.0106
3	$s_1, s_2, Tstat$	0.00708
4	$s_1, s_3, s_4, Tstat$	0.22
5	$s_1, s_3, s_4, s_2, Tstat$	-



Zone Temperature – Business as usual



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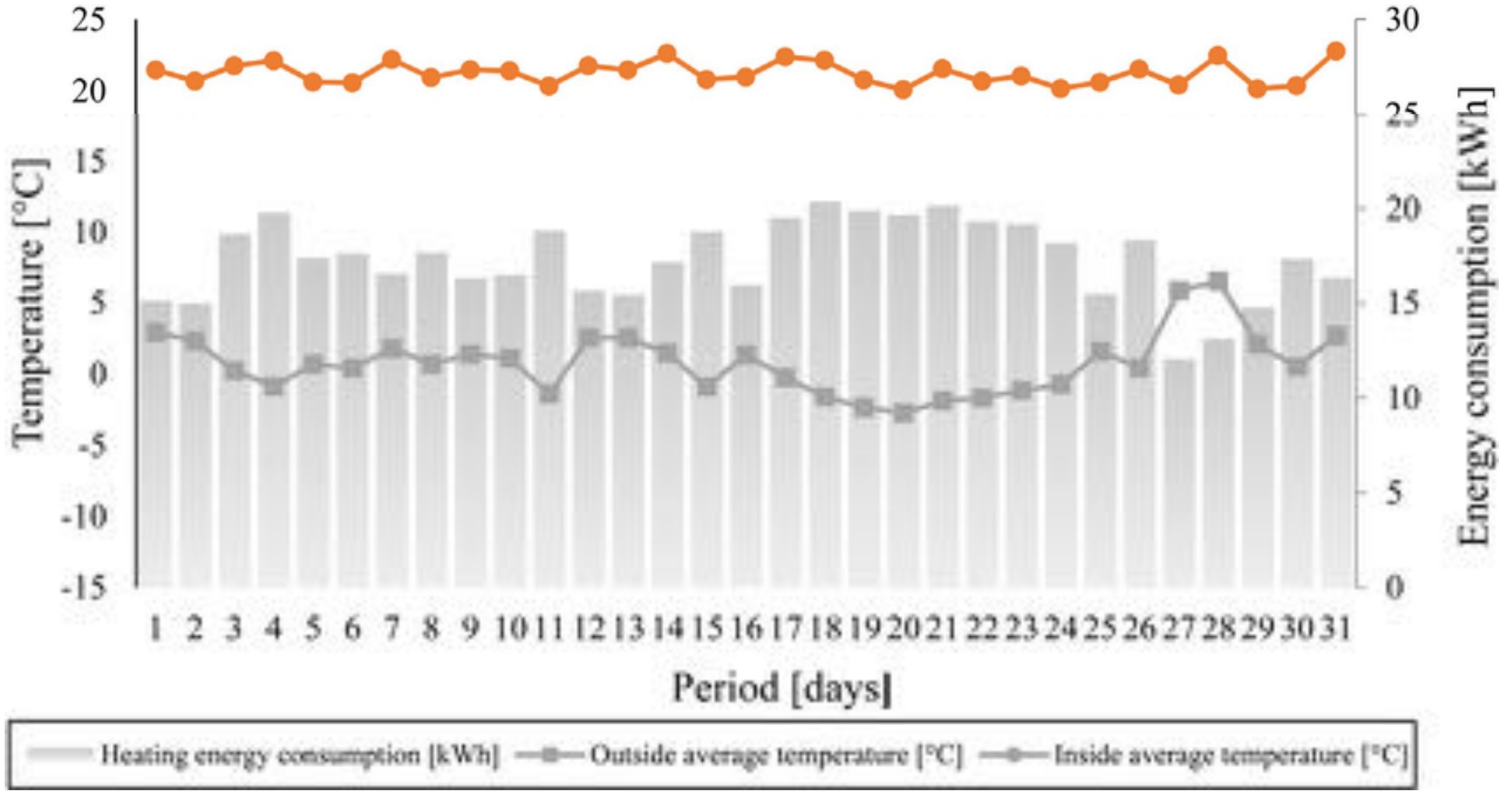
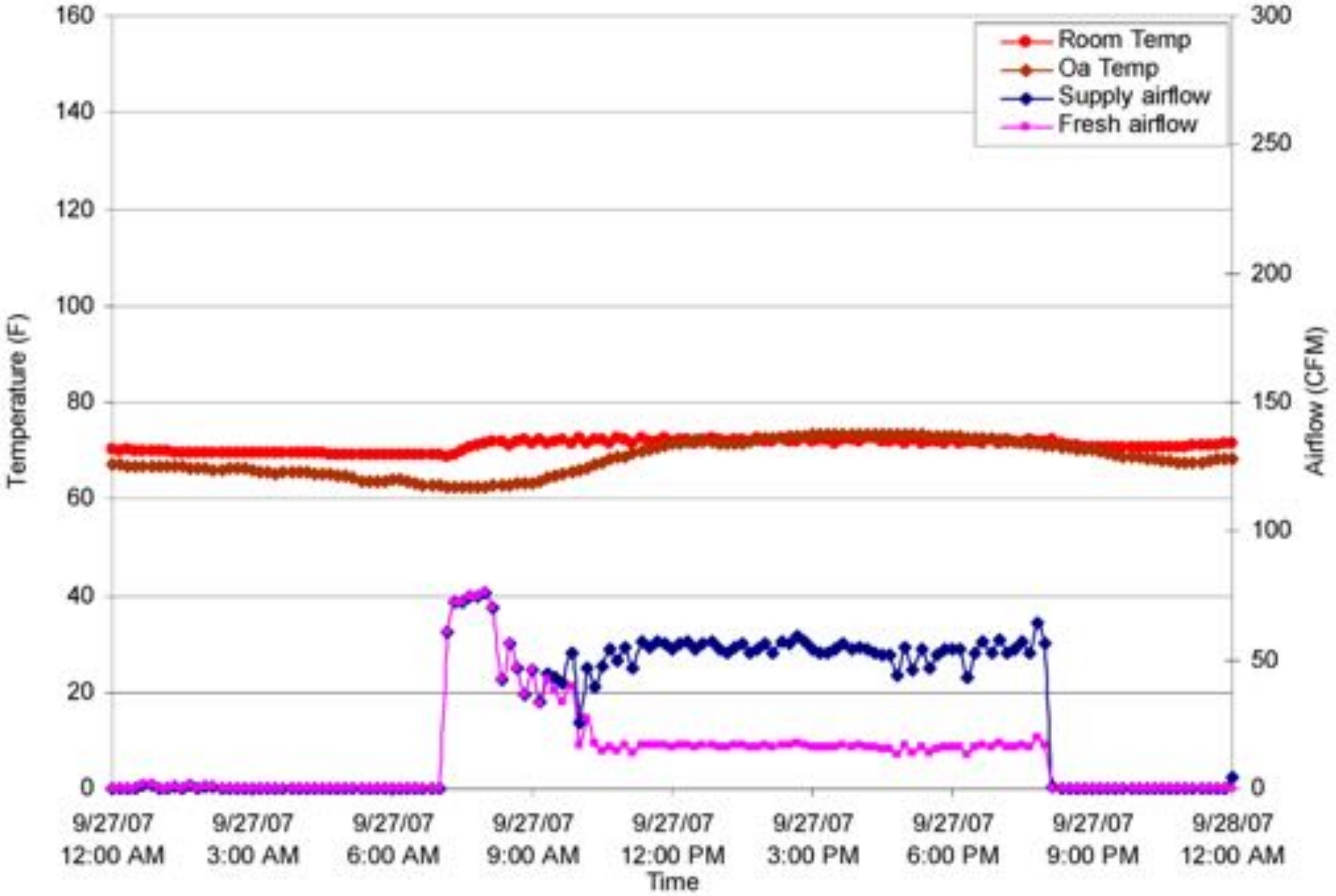
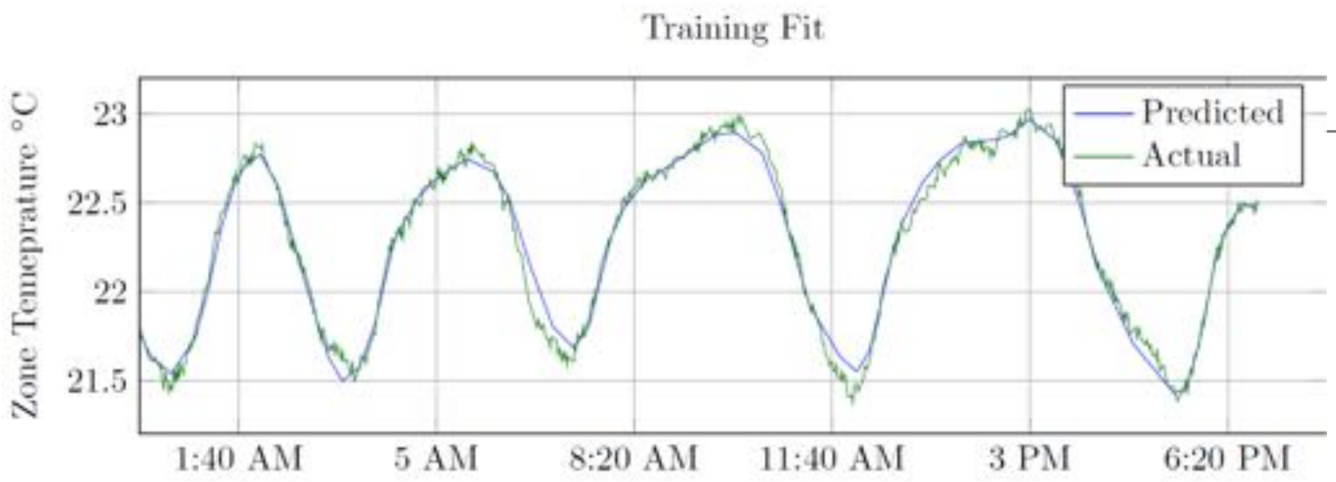


Zone Temperature – Business as usual



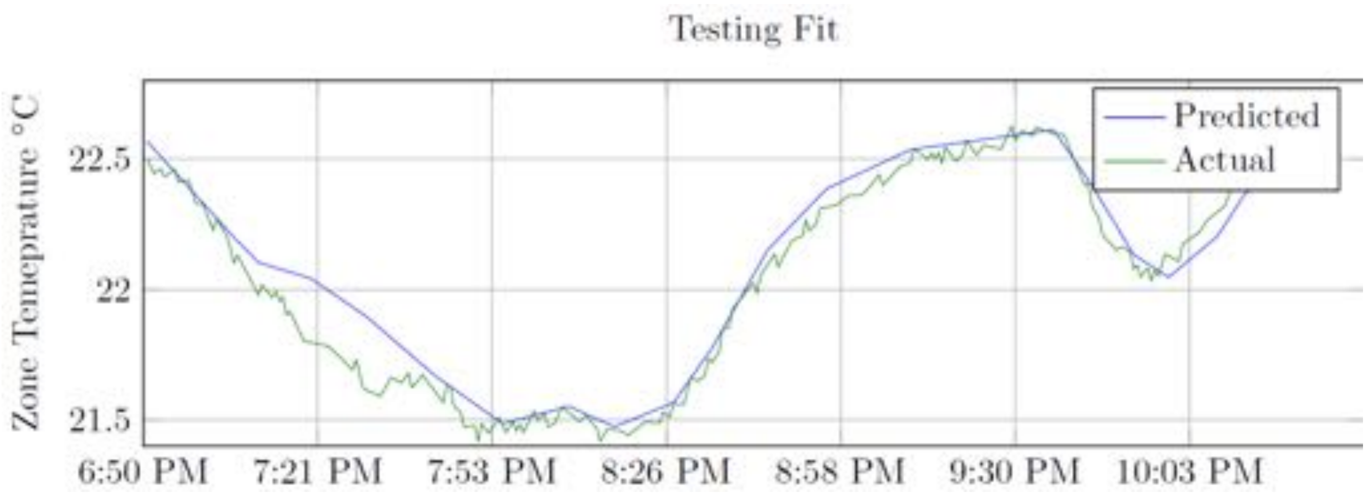
Case study: Building 101



Model Accuracy for Training data

RMSE: 0.062 °C
R2: 0.983

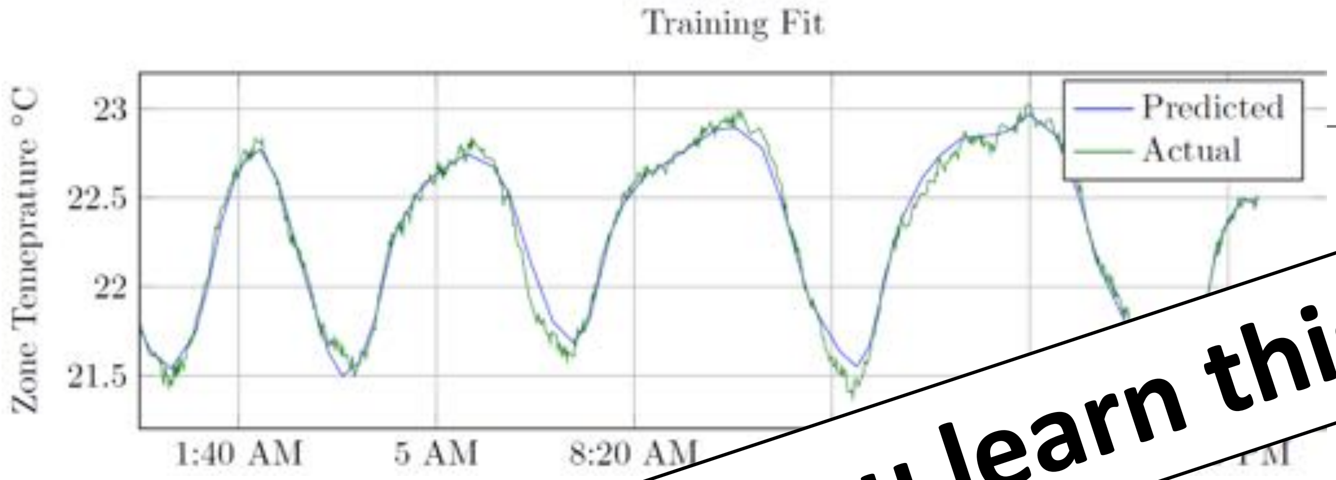
Baseline



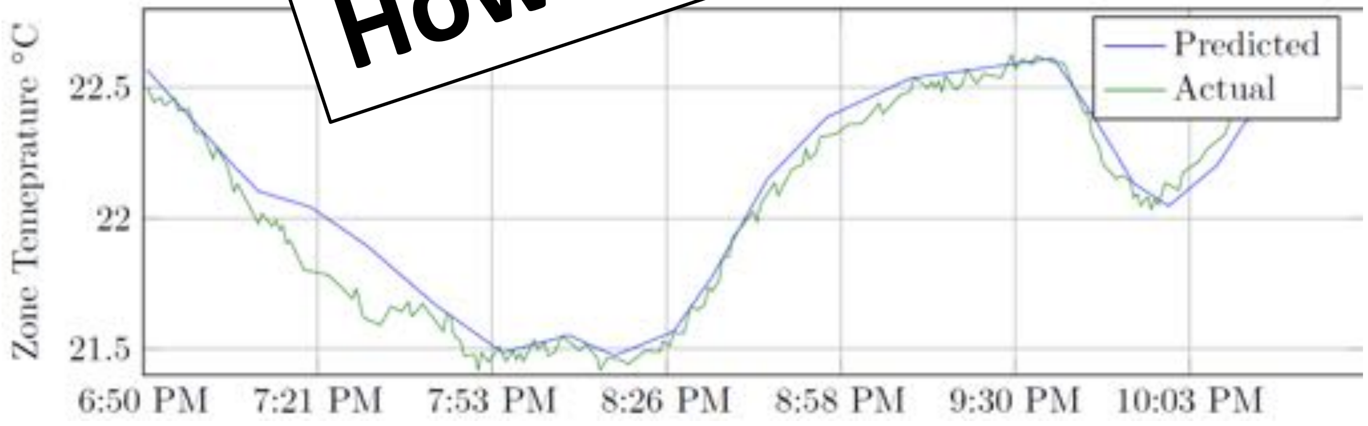
Model Accuracy for Test Data

RMSE: 0.091 °C
R2: 0.948

Case study: Building 101



How did you learn this model ?



Model Accuracy for T[illegible]g data

[illegible].062 °C
R2: 0.983

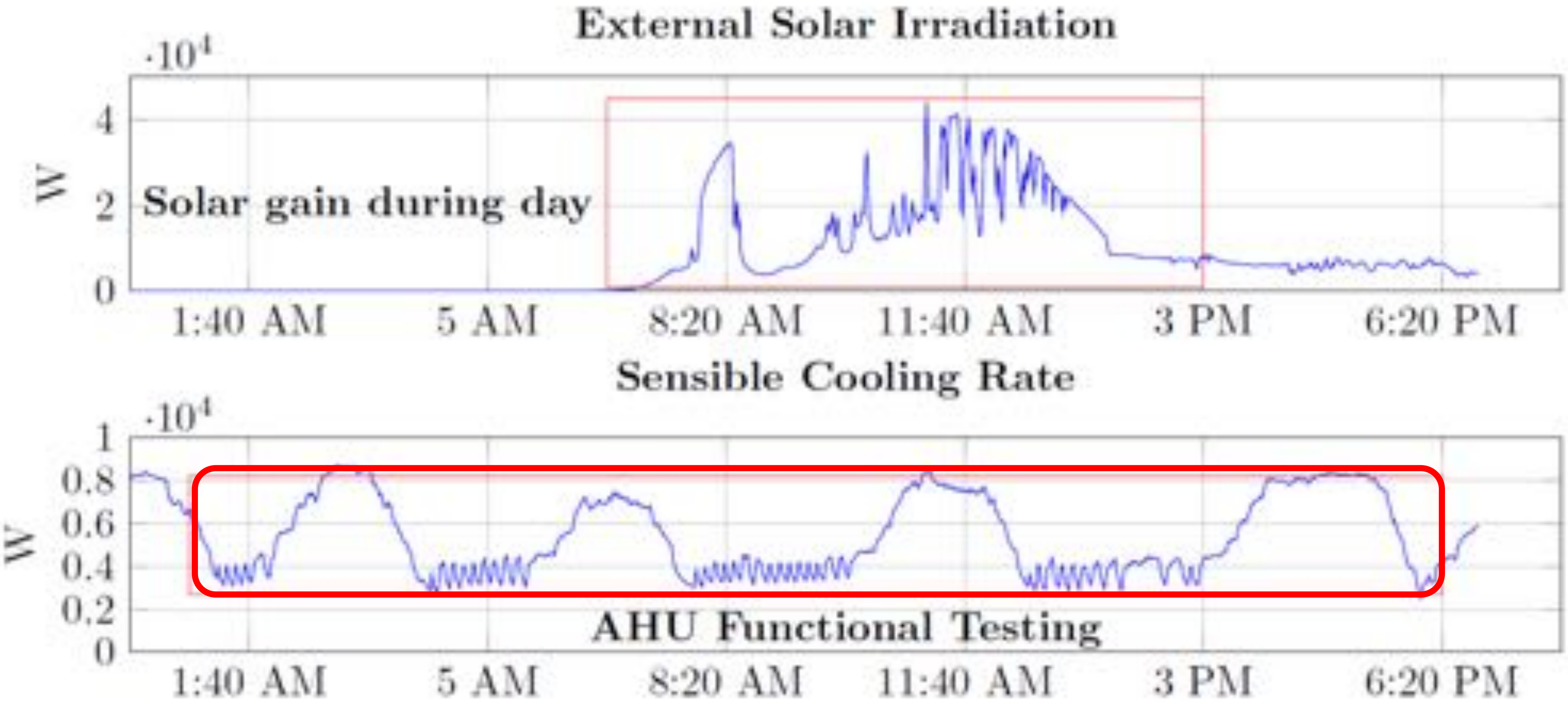
Baseline

Model Accuracy for Test Data

RMSE: 0.091 °C
R2: 0.948

AHU Functional Tests: Suite 210

Functional tests were carried out in Suite 210 in June 2013.



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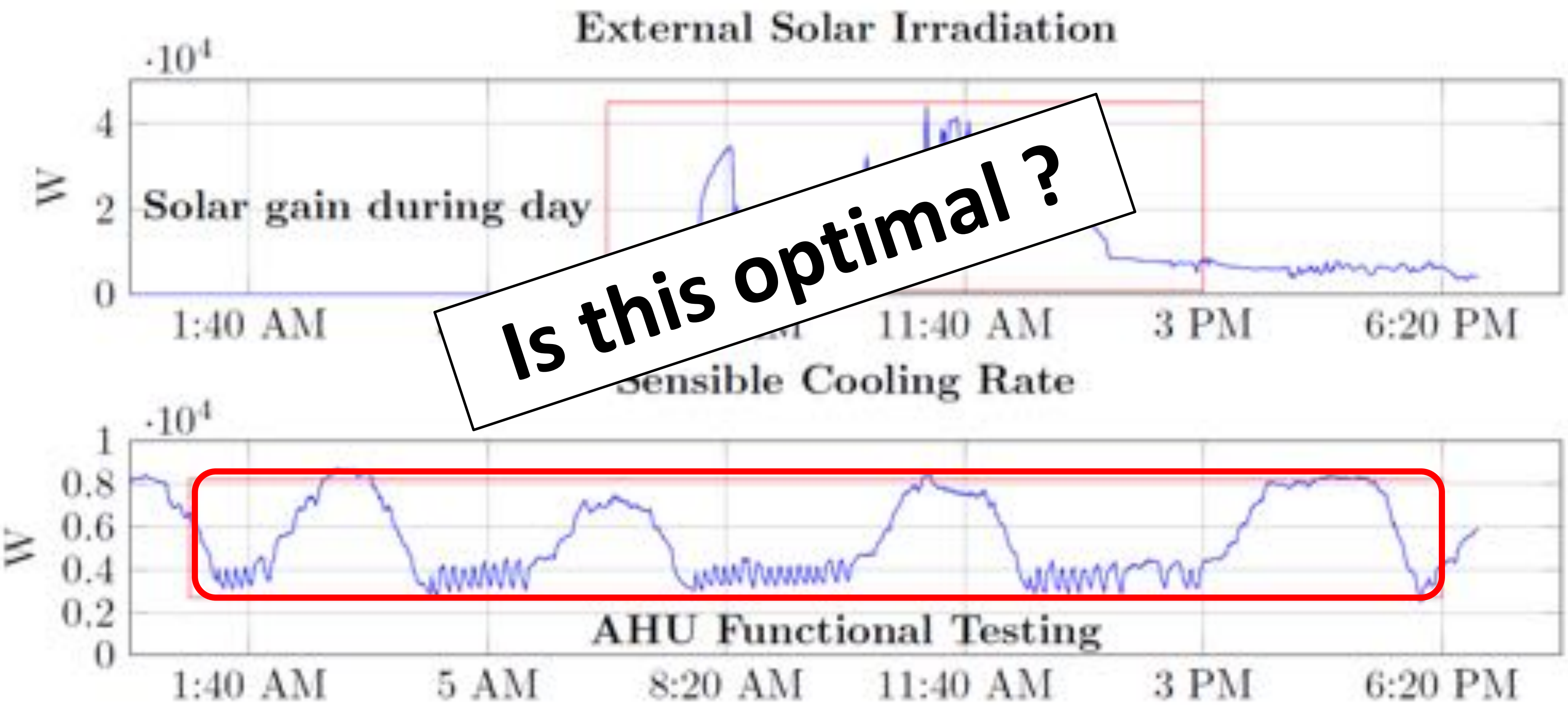

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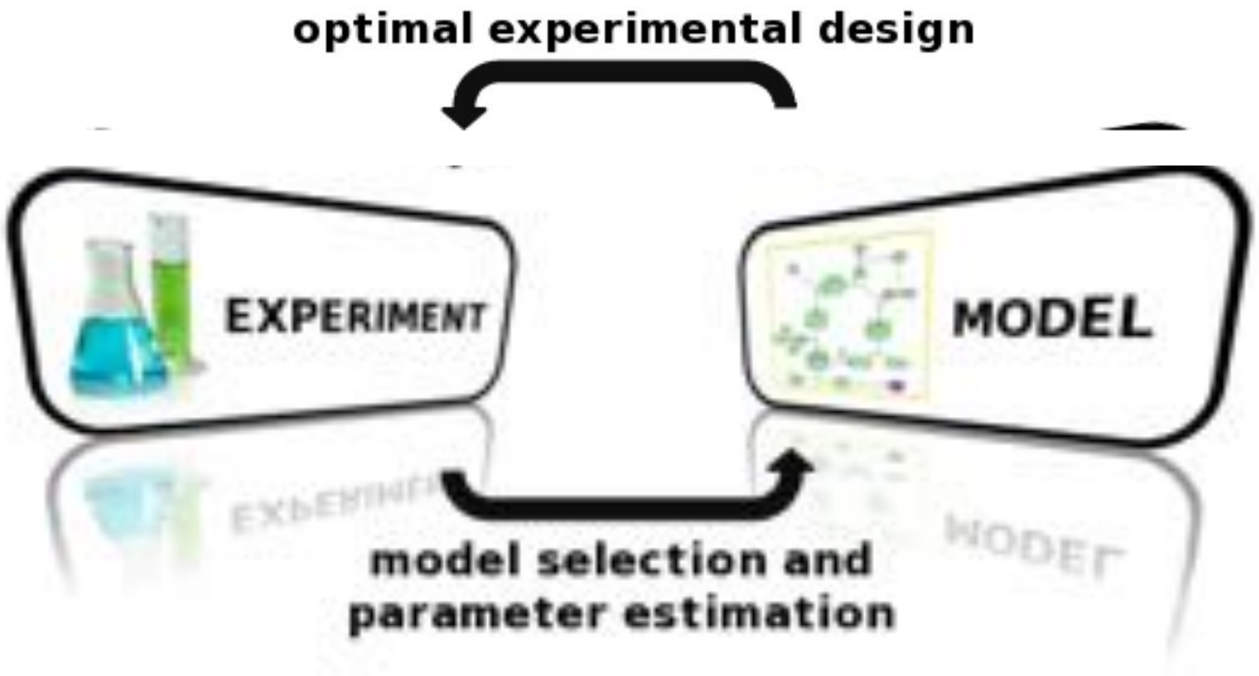
What is Experiment Design ?



Optimal Experiment Design

Find the optimal input signal trajectory which maximizes the information about the model parameters subject to operational constraints.

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Optimal Experiment Design

Requires some optimality criteria

Find the **optimal** input signal trajectory which maximizes the **information** about the **model parameters** subject to **operational constraints**.

Specify information metric

Cannot play with set-points at the cost of comfort

Needs a suitable parameter estimation method

Maximum Likelihood & Fisher Information

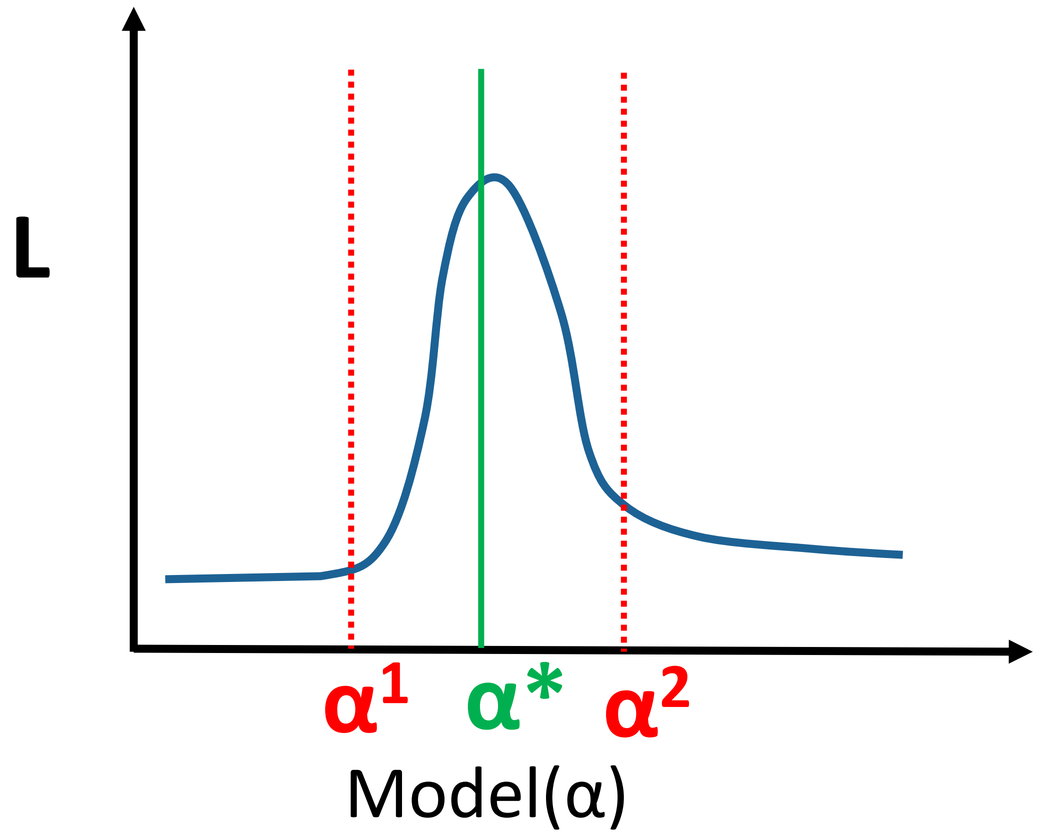
Likelihood functions play a key role in statistical inference and parameter estimation.

$$L = \mathbf{P}\left[\texttt{data} \middle| \texttt{model}\right]$$

The probability that we see the given data due to the model we have assumed for the building/equipment.

Maximum Likelihood & Fisher Information

$$L = \mathbf{P}[\texttt{data}|\texttt{model}]$$



True parameter value

Inconsistent parameter values

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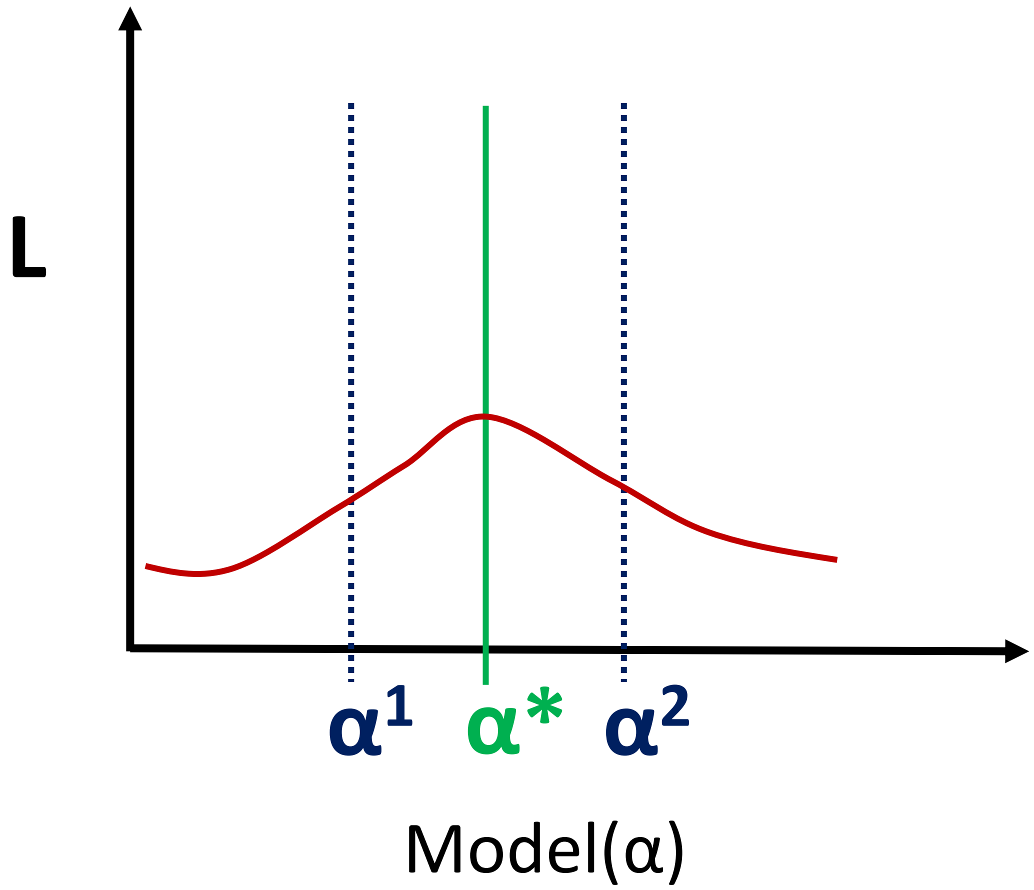
Maximum Likelihood & Fisher Information

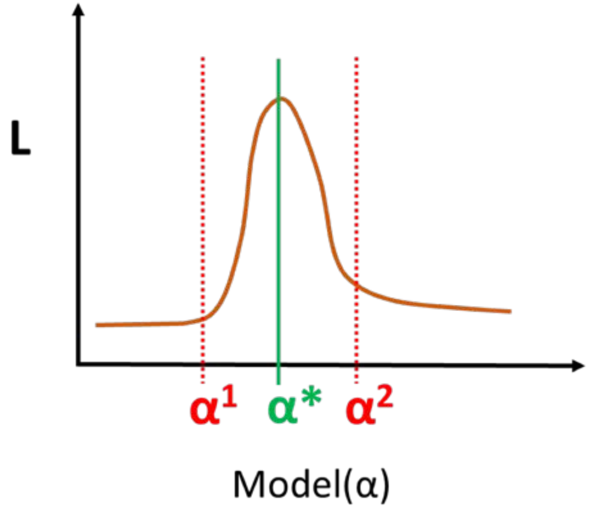
$$L = \mathbf{P}\left[\mathtt{data}\middle|\mathtt{model}\right]$$



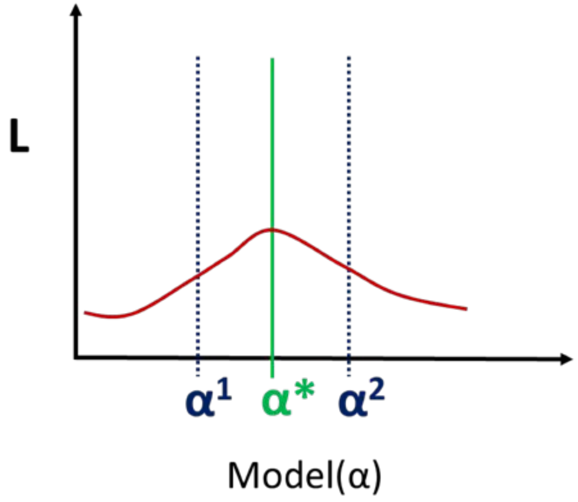
True parameter value

Consistent parameter values

Maximum Likelihood & Fisher Information



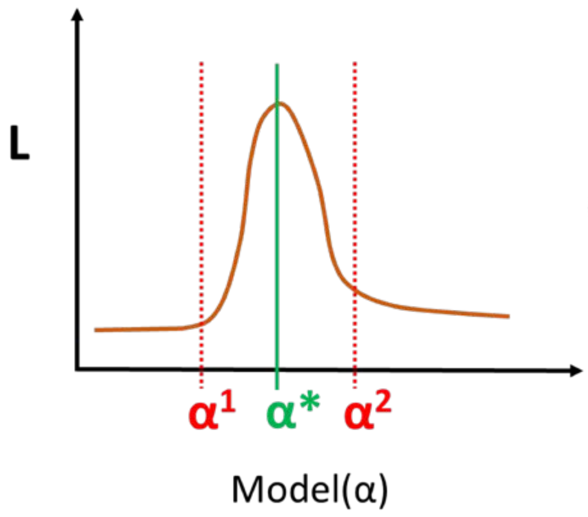
$$L = \mathbf{P}[\texttt{data}|\texttt{model}]$$



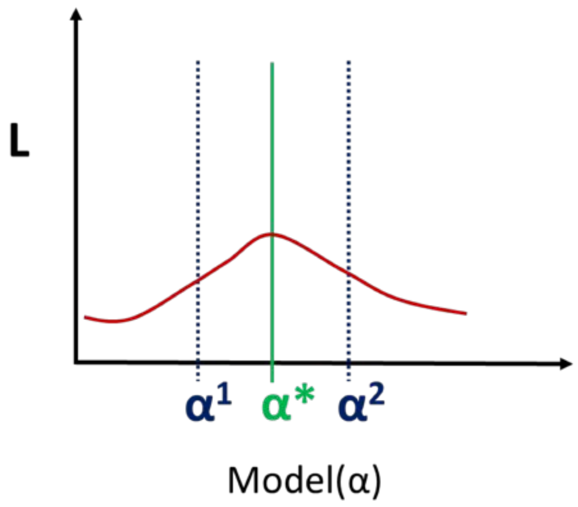
(1) We want an estimate which maximizes the likelihood function.

Maximum Likelihood Estimate

Maximum Likelihood & Fisher Information



$$L = \mathbf{P}[\mathtt{data}|\mathtt{model}]$$



(2) Some way to quantify the difference between likelihood functions i.e. how quickly does it fall of around the maximum

$$L(\alpha) \equiv L(\alpha) + \frac{\partial L(\alpha)}{\partial \alpha}|_{\alpha^*}(\alpha - \alpha^*) + \frac{\partial^2 L(\alpha)}{\partial \alpha^2}|_{\alpha^*}(\alpha - \alpha^*)^2$$

= 0 at maxima α*

Fisher information

Cramer-Rao bound

Let $\mathbf{y}^t$ denote the set of t measurements y(0), y(1), ……, y(t-1).

The likelihood function $L(\alpha) = \mathbf{P}[y^t; \alpha]$

model (α)

data (y^t)

For any <u>unbiased</u> estimator we have the following Cramer-Rao lower bound:

$$\mu(\alpha) = \alpha^* - \mathbf{E}[\hat{\alpha}|\alpha^*]$$

$$\boxed{\Sigma(\alpha) \geq I^{-1}(\alpha)}$$

Error covariance of α

$$\Sigma(\alpha) = \mathbf{E}[(\alpha^* - \hat{\alpha})(\alpha^* - \hat{\alpha})'|\alpha^*]$$

Fisher information matrix (FIM)

$$I_{y^t}(\alpha) = -\mathbf{E}\left[\frac{\partial^2}{\partial \alpha^2} ln\mathbf{P}(y^t|\alpha)\right]$$

For the RC 'grey box' building model

$$x(t+1) = A_\alpha x(t) + B_\alpha u(t) + W\omega(t)$$
$$y(t+1) = C_\alpha x(t+1) + D_\alpha u(t+1) + \nu(t+1)$$

State space model

$$\mathbf{P}[y(\tau)|y(\tau-1);\alpha] = \frac{1}{\sqrt{2\pi \det[F(t)]}} e^{\frac{-1}{2}[r^T(t)F^{-1}(t)r(t)]}$$

Likelihood function

Need Kalman filter equations to compute the likelihood function.

$$F(t) = \mathbf{E}[y(t) - \hat{y}(t|t-1)][y(t) - \hat{y}(t|t-1)]^T$$
$$= C\Sigma(t|t-1)C^T + R$$

$$\Sigma = A\Sigma A^T + WQW^T - A\Sigma C^T(C\Sigma C^T + R)^{-1}C\Sigma A^T$$

But where is the experiment design ?

First we compute the Fisher Information Matrix

$$\underline{I}_{\underline{z}^t}(\underline{\alpha}) = \sum_{\tau=0}^{t} \mathrm{tr}\left[\overline{\frac{\partial \underline{r}(\tau;\underline{\alpha})}{\partial \alpha_i} \frac{\partial \underline{r}'(\tau;\underline{\alpha})}{\partial \alpha_j}}\, \underline{S}^{-1}(\underline{\alpha})\right]$$

$$+ (t+1)\ \mathrm{tr}\left[\underline{S}_{ij}(\underline{\alpha})\underline{S}^{-1}(\underline{\alpha}) + \tfrac{1}{2} \frac{\partial \underline{S}(\underline{\alpha})}{\partial \alpha_i} \underline{S}^{-1}(\underline{\alpha}) \frac{\partial \underline{S}(\underline{\alpha})}{\partial \alpha_j} \underline{S}^{-1}(\underline{\alpha})\right]$$

Depends only on the inputs and disturbances into the system

Optimality criteria

Optimality criteria of the information matrix

- A-optimal design ⇔ average variance

$$\min_{y} trace(I(y)^{-1})$$

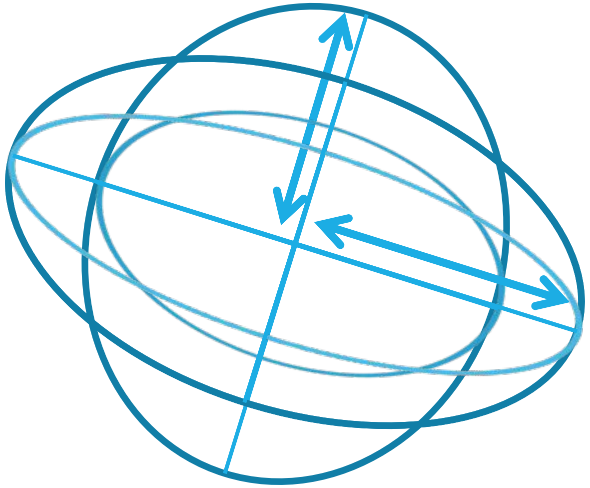
- D-optimality ⇔ uncertainty ellipsoid

$$\min_{y} det(I(y)^{-1})$$

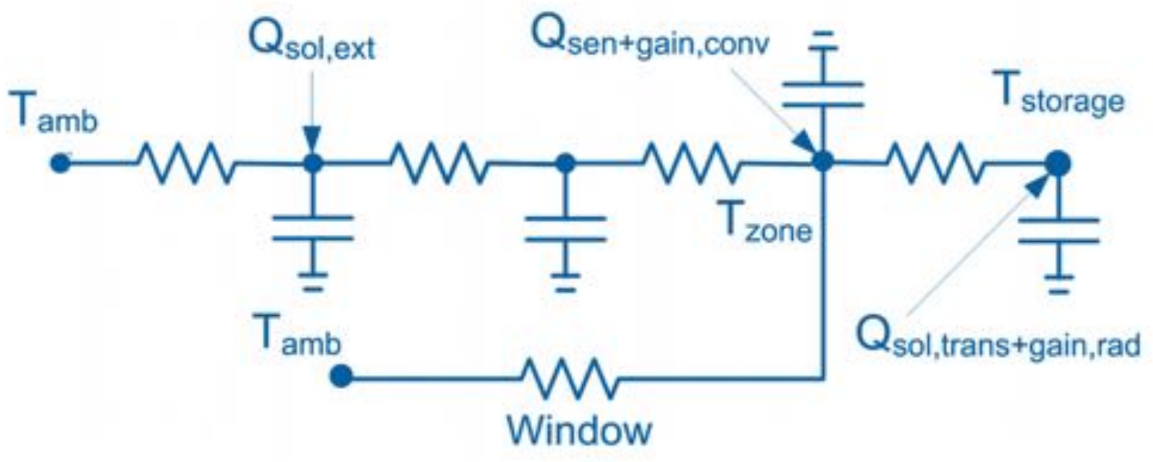
- E-optimality ⇔ minimax

$$\min_{y} \max(eig(I(y)^{-1}))$$

- Almost a complete alphabet…



Example



Input:

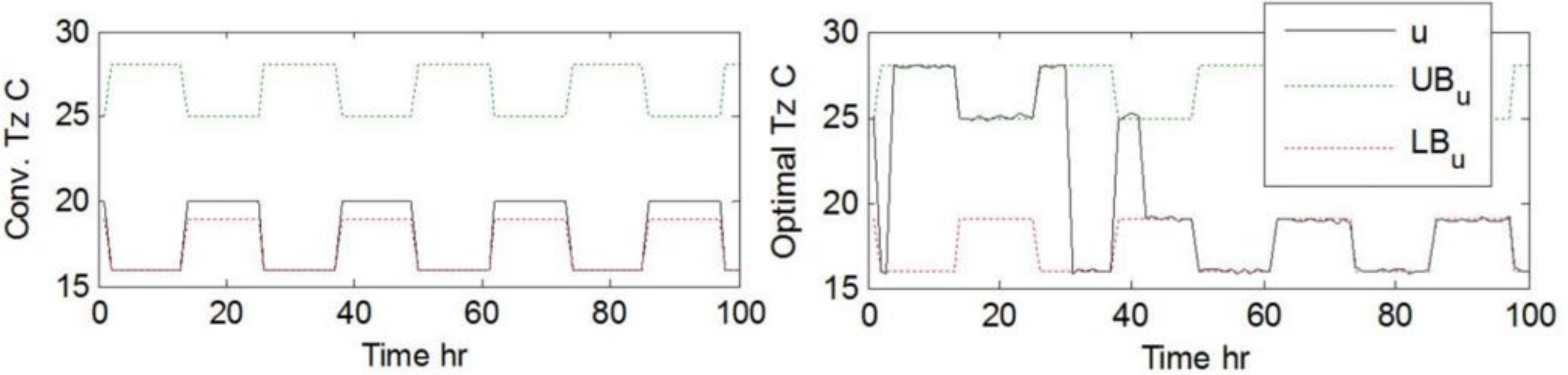
T_{amb}, $T_{storage}$, Q_{sol}, Q_{conv}, $Q_{sol,trans}$,Q_{rad}, T_{zone}

Output:

Q_{sen}

Cai, Jie, et al. "Optimizing zone temperature set-point excitation to minimize training data for data-driven dynamic building models." *American Control Conference (ACC), 2016*. IEEE, 2016.

Optimal and conventional temperature set-point profiles



Performance comparison of models trained with conventional and optimal training data sets