System:

$$x_{k+1} = Ax_k + Bu_k + \varepsilon_k$$

$$\boxed{y_k = Cx_k + Du_k + v_k}$$

$$\varepsilon \equiv N(0, R)$$

$$v \equiv N(0, Q)$$

$$x_0 \equiv N(\_\,,\ \_)$$

$$x_{k|k} = p(x_k \mid y_1 \,\dots\, y_k)$$

$$\equiv N(\mu_{k|k}, \Sigma_{k|k})$$

$$\underset{(\mu_{k|k},\, \Sigma_{k|k})}{x_{k|k}} \xrightarrow[\text{propagation}]{\text{Step 1}} x_{k+1|k} \xrightarrow[\text{Step 2}]{\text{update}} x_{k+1|k+1}$$

$$\hookrightarrow p(x_{k+1} \mid y_1 \cdots y_{k+1})$$

$$=$$

Step 1 $\left(\mu_{k+1|k}, \Sigma_{k+1|k}\right) \equiv x_{k+1|k}$

$$x_{k+1|k} = A x_{k|k} + B u_k + \varepsilon_k$$

$$E(x_{k+1|k}) = E(A x_{k|k} + B u_k + \varepsilon_k)$$

$$\mu_{k+1|k} = A \mu_{k|k} + B u_k$$

$$\Sigma_{k+1|k} = A \Sigma_{k|k} A^T + R$$

Step 2 : $\hat{x} + y$

Step 2 Given $x_{k+1|k} \equiv N(\mu_{k+1|k}, \Sigma_{k+1|k})$

$\mu_{k+1|k+1}, \Sigma_{k+1|k+1} \equiv x_{k+1|k+1}$

$$y_{k+1} = C x_{k+1} + v_{k+1} \quad [D=0]$$

$$\underline{\mu_{k+1|k+1}} = \mu_{k+1|k} + \mathbb{K}(y_{k+1} - C\mu_{k+1|k})$$

$$\mathbb{K} = \Sigma_{k+1|k} C^T (C \Sigma_{k+1|k} C^T + Q)^{-1}$$

$$p(x_{k+1} | y_1 \cdots y_{k+1}) \equiv N(\mu_{k+1|k+1}, \Sigma_{k+1|k+1})$$

$$\underbrace{\Sigma_{k+1|k+1}} = \left(\Sigma_{k+1|k}^{-1} + C^T Q^{-1} C \right)^{-1}$$

"inverse covariances add together"

$$\Sigma_{new}^{-1} = \Sigma_{old}^{-1} + \text{noise}^{-1}$$

$$\begin{tikzcd}
x_{k|k} \arrow[r, "1"] & \underbrace{x_{k+1|k}} \arrow[r, bend left, "\text{update } 2"] & x_{k+1|k+1} \arrow[r] & x_{k+2|k+1} \arrow[r, bend left] & F_{k+2}
\end{tikzcd}$$

Camera

$\downarrow$

10 Hz

LIDAR

$\downarrow$

50 Hz

U(k)

100 Hz

$x^0_{k|k} \longrightarrow x_{k+1|k} \longrightarrow x_{k+C|k+1} \longrightarrow \cdots$

C L L L C

$1/f$

$\mathbb{K}_C$ $\mathbb{K}_L$