

System:

$$x_{k+1} = Ax_k + Bu_k + \varepsilon_k$$

$$y_k = Cx_k + Du_k + v_k$$

$$\varepsilon \equiv \mathcal{N}(0, R)$$

$$v \equiv \mathcal{N}(0, Q)$$

$$x_0 \equiv \mathcal{N}(\dots)$$

$$x_{k|k} = p(x_k | y_1 \dots y_k)$$

$$\equiv \mathcal{N}(\mu_{k|k}, \Sigma_{k|k})$$

Step 2

update

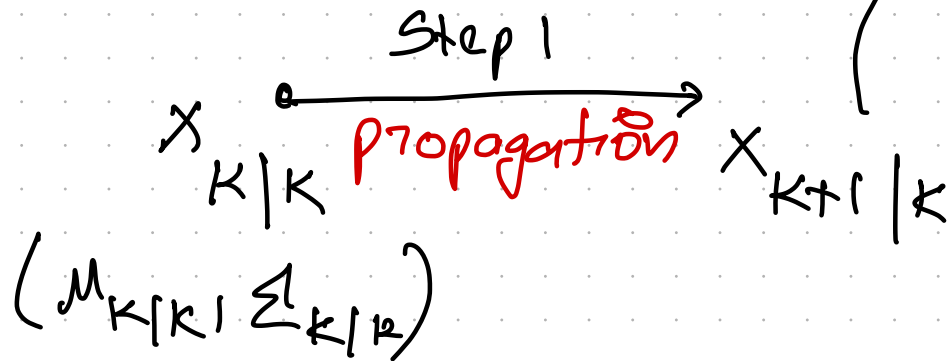


Diagram illustrating Step 2: Update.

A curved arrow labeled "update" in red points from  $x_{k+1|k}$  to  $x_{k+1|k+1}$ . Below  $x_{k+1|k+1}$  is the expression  $p(x_{k+1} | y_1 \dots y_{k+1})$ .

Step 1  $(\mu_{k+1|k}, \Sigma_{k+1|k}) \equiv x_{k+1|k}$

$$x_{k+1|k} = Ax_{k|k} + Bu_k + \varepsilon_k$$

$$E(x_{k+1|k}) = E(Ax_{k|k} + Bu_k + \varepsilon_k)$$

$$\mu_{k+1|k} = A\mu_{k|k} + Bu_k$$

$$\Sigma_{k+1|k} = A\Sigma_{k|k}A^T + R$$

Step 2  $\hat{x} + y$

Step 2 Given  $X_{k+1|k} \equiv \mathcal{N}(\mu_{k+1|k}, \Sigma_{k+1|k})$

$\mu_{k+1|k+1}, \Sigma_{k+1|k+1} \equiv X_{k+1|k+1}$

$$y_{k+1} = C X_{k+1} + v_{k+1} \quad [D=0]$$

$$\underline{\mu_{k+1|k+1}} = \mu_{k+1|k} + K (y_{k+1} - C \mu_{k+1|k})$$

$$K = \Sigma_{k+1|k} C^T (C \Sigma_{k+1|k} C^T + Q)^{-1}$$

$$P(\underline{X_{k+1}} | y_1, \dots, y_{k+1}) \equiv \mathcal{N}(\mu_{k+1|k+1}, \Sigma_{k+1|k+1})$$

$$\Sigma_{k+1|k+1} = \left( \Sigma_{k+1|k}^{-1} + C^T Q^{-1} C \right)^{-1}$$

"inverse covariances add together"

$$\Sigma_{\text{new}}^{-1} = \Sigma_{\text{old}}^{-1} + \text{noise}^{-1}$$

